HINTS AND ANSWERS.

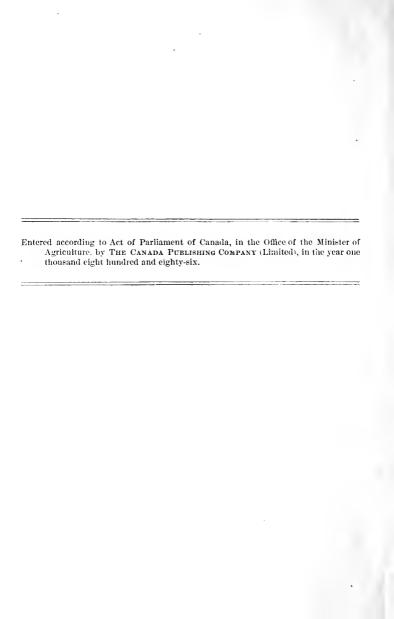## McLELLAN'S
Elements of Algebra.

HINTS AND ANSWERS

TO THE EXERCISES

IN

ELEMENTS OF ALGEBRA.

BY

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ANSWERS.

EXERCISE XIV [b]. (Page 32.)

1. $6x^2 - 12x^3y + 7xy^2 - 3y^3 - y^2 + 2xy^3$.

2. $-8(m+n) + 5(a+b)$.   3. 0.   4. $8(m+n)^2 - y$.

5. $x^3 + x^2y + 7xy^2 + 2y^2 + y^3$.   6. $\frac{3}{2}a^2 - a^3 - 2a^2b + \frac{1}{4}ab^2 + b^2$.

7. $10x \div y - 13m \div n$.

EXERCISE XV.

1. $(3 + 6b + 7a)x + (-2 - 4)y + m + n$.

2. $(a + m + 1)x + (1 + n - d)y$.

3. $(6a - 3b - 2)x + (1 + \frac{3}{4}b + \frac{2}{3}a)y$.

4. $(2d - 2f)x + (3e - 3d)y + (4f + 4e)z$.

5. $(a + b - 4)x + (\frac{1}{2}a^2 + c^2 - \frac{2}{3}a^3 - 6)y$.   6. $3ax - 3by$.

7. $(a - 6)x + (5m + 5)\sqrt{y} + (b - 1)y - 3\sqrt{x}$.

8. $(a - c)x^2 + (b - a)y^2 + (c - b)z^2 + ax + by + cz$.

9. $(1 - 9a - 2b - c)x^n + (1 + 7b + 10a - 3abc)y^n$.

EXERCISE XVI [b]. (Page 34.)

1. $-9x^4 + 12x^3y + 6x^2y^2 - 18xy^3 + 21y^4 + 20$.

2. $p^2 - 17q^2 - 22r^2 + 17pq - 8yz^2 + 99$.

3. $-3(x - y) + 34(x - z)$.   4. $19(a - b) - 8(a + b) + 14a + b$.

5. $12\dfrac{x}{y} - 11\dfrac{y}{x} - 8\dfrac{z}{x} - 8\dfrac{a}{b} - 6\dfrac{y}{z}$.

6. $2x^2 - \frac{1}{3}xy - \frac{1}{2}y^2$.   7. $-\frac{1}{8}a^3 - \frac{2}{3}a^2x - \frac{3}{2}ax^2$.

8. $-2x^3 + 3xy^2 - y^3 - 14x^2 + 2xy - 10y^2 + 2$.

9. $a^2b^2 - 3a^2bc - 3ab^2c - a^2c^2 - abc^2 - b^2c^2$.   10. $a^4 - 2a^2b^2 + b^4$.

11. $(a^2 - b^2)x^2 + (b^2 + c^2)y^2 + (c^2 - a^2)z^2$.

12. $19a^m - 17b^n - 2c^p + 10d^q$.

13. $(a - p)x^3 + (q - b)x^2 + (1 - r)x + 1$.

14. $a^3 + a^2b + 6ab^2 - 2b^3 + 3b^2$.   15. $\frac{1}{2}y - 1\frac{3}{6}a - \frac{3}{4}x$.

16. $-\frac{9}{7}(xyz - bx + cy)^2 - 4\frac{3}{10}(z - y + ax)$.

17. *Sum* is $37\frac{3}{4}a - 12\frac{13}{24}b - 4\frac{17}{24}c - 7\frac{1}{6}d - 28\frac{1}{2}e$; the several remainders are $35\frac{1}{4}a - 9\frac{7}{24}b - 11\frac{11}{24}c - 1\frac{5}{6}d - 24\frac{1}{4}e$;
$33\frac{11}{12}a - 11\frac{5}{8}b - 5\frac{23}{24}c - 6\frac{1}{12}d - 20\frac{11}{12}e$;
$31\frac{2}{3}a - 12\frac{7}{8}b - 3\frac{5}{24}c - 5\frac{1}{4}d - 22\frac{1}{8}e$;
$29\frac{1}{8}a - 8\frac{1}{8}b + 3\frac{1}{8}c + 1\frac{7}{8}d - 17e$;
$22\frac{5}{12}a - \frac{2}{8}b - 4\frac{1}{4}c - 4\frac{11}{24}d - 12\frac{3}{8}e$;
$16\frac{2}{3}a + 2\frac{7}{8}b - 1\frac{11}{12}c - 1\frac{1}{12}d - 7\frac{1}{2}e$;
$9\frac{1}{3}a + 8\frac{1}{2}b + 4\frac{3}{8}c + 6\frac{2}{3}d - 4\frac{1}{3}e$; 0.

18. i. 0; a; $\sqrt[3]{(3a^3)}$.   ii. $26a^6$; $a + x$.

EXERCISE XVII [b].   (PAGE 39.)

1. 6, 18.
2. 30, 40.
3. 35, 65.
4. 4, 2, 10.
5. 8, 40, 12.
6. 500.
7. 120, 137, 163.
8. 120.
9. 30 minutes.
10. \$840.
11. 7, 42.
12. \$32, \$36, \$44.
13. 24.
14. \$138\frac{8}{9}, \$236\frac{1}{9}.
15. £1200.
16. 7 months.
17. 450, 180, 140.
18. 300.
19. \$100.
20. $12\{22x - \frac{5}{11}(20 - 33x)\} = 44x$, $x = \frac{3}{11}$.

EXERCISE XVIII.   (PAGE 41.)

1. $a^2 + b^2 - c^2 - d^2$; $a^2 - b^2 + c^2 + d^2$.
2. $a^2 - 3b^2 + c^2$.
3. $2m - n + 6$.
4. $-2x - 3y - 2z$.
5. $1\frac{1}{3}x - 4\frac{5}{6}y + 1\frac{1}{5}z$.

EXERCISE XIX.   (PAGE 43.)

1. $-2a + 3x + 3b$.
2. $a + b + c$.
3. $2ab + 4b^2$.
4. $-3x - y + 4z$.
5. $5 - 4x$.
6. $2a - 3b - 3c + 4d$.
7. $-4a$.
8. $-x - 10y + 2z$.
9. $-2x + 2y$.
10. $2x - 6y - my + 4ab - 5$.
11. $3a - 5b - c$.
12. 0.
13. $-\frac{1}{6}y$.
14. $\frac{4}{5}a - 2b$.
15. $\frac{1}{2}x$.
16. 9.

EXERCISE XX.   (PAGE 44.)

1. i. $x - (a + b)$; $x - (a + 3b - 2y)$.
   ii. $x - (2m - 2n)$; $x - (3b - 2c - 5d)$.
   iii. $x - (2m + 3a - 2b)$; $x - (b - a - c - m + n)$.
   iv. $x - (a + b - c - 12)$; $x - \{(a + b) + (p + q) + (m - n)\}$.

2. i. $(2a - 4b - 3c) x - (6a + 3c) y + (4b - ac) z.$

ii. $(a - b + c) x - (a + b - c) y - (a - b - c) z.$

iii. $(12a - 15c) x - (12a + 4b + 6c) y - (12b + 3c) z.$

3. i. $2 + (7 - 2c) x + (5a - 3) x^2 + (9a - 7) x^3.$

ii. $(2c - a^2) x^5 + (a - 3b) x^4 + (1 - m) x^3 + (4c - 3ab) x.$

iii. $(1 - a) x^4 + (1 - b + c) x^3 + (b - 1) x^2 + (a - 7) x + 2.$

4. i. $- (3c^2 - 5a) x - (abc - 7) x^3 - (ab - 7) x^5.$

ii. $1 - (a - 1) x - (1 - b) x^2 - (a - c + 1) x^3 - (a - b - 1) x^4.$

iii. $- (a - 3b^2) x^4 - (1 - c) x^3 - (1 + 5c^2) x^2 - (b + c) x.$

5. i. $(a - c + 1) x^3 - (a + 2b + 1) x^2 + (b + c) x + 3.$

ii. $(5a + 4c) x^3 + (7c - 6b + 3a) x^2 + (2a - 7b) x.$

iii. $(a - b + c) x^2 - 2 (a + b + c) x + ab - bc - ca.$

6. i. $6 ; 6.$ ii. $- 17 ; - 9.$ iii. $- 1 ; - 56.$

7. $(a + b + c) (x + y + z).$

8. $- 3a - rx - (2 - b) x^2 + (4a - p - 1) x^3 + 2x^4.$

9. $(6y + 1) x^5 - (z + 2y) x^4 - (2z + 3) x.$

EXERCISE XXI [b]. (PAGE 48.)

1. $36 ; - 48 ; 5 ; 9 ; - 168 ; - 180.$

2. i. $m^3xyz ; abcx^3 ; - 24a^5b^3.$ ii. $- 36a^6m^4 ; - a^3b^3c^3x^2y^2z^3.$

iii. $- 14a^2b^2x^2 ; - 18x^3y^2z^4 ; - 5x^3y^2z^2.$

3. i. $40 ; - 63 ; - 2 ; - 37.$ ii. $130 ; - 880 ; 0.$ iii. $\frac{1}{2} ; 29.$

EXERCISE XXII [b]. (PAGE 49.)

1. $a^3b^2c - ab^4 + abc^3 ; - \frac{5}{2}x^2 + \frac{5}{3}xy + \frac{10}{3}x.$

2. $3abxy + 6acxz + 15ax ; 9x^4yz^2 - 12x^2y^3z^2 + 15x^2yz^4.$

3. $- 15x^5y - 10x^4y^2 + 35x^3y^3 - 5x^2y^4 ; 3a^4 + 2a^5b - a^6b^2.$

4. $3x^3y^2z - 3x^2y^4z + 3x^2y^2z^4 - 12x^4y^4z^3 ; \frac{1}{4}a^2x - \frac{1}{16}abx - \frac{3}{8}acx.$

5. $- 2a^5x^3 + \frac{7}{2}a^4x^4 + a^6c^4 ; - x^5y^5 + \frac{16}{43}c^5y^6.$

6. $\frac{3}{4}x^4y^2z^2 - \frac{3}{2}x^3y^3z^2 + \frac{3}{4}x^2y^4z^2 - x^3y^3z^3 ; \frac{5}{4}a^4x^2 - \frac{5}{3}a^2x^3 + a^2x^4.$

7. $- 2 (a + b)^4 + 2 (a + b)^2 ; - 3 (a - b)^3 - 2 (a - b)^5.$

8. $(m^2 - n)^7 + (m^2 - n)^5 ; 3 (a + b)^{n-1} + 2 (a + b)^{n+4}.$

9. $(a + b)^{n+1} + (a + b)^{m+1} ; (a - b)^{n+1} - (a - b)^{n+2}.$

EXERCISE XXIII. (PAGE 50.)

1. $6x^2 - 13xy + 6y^2$. 2. $15x^3 - 3x^2y - 5bx + by$.

3. $x^3 - 9a^2x$. 4. $-10b^4 - 15ab^3 + 14ab^2 + 21a^2b$.

5. $a^3 + b^3$. 6. $a^2 - b^3$. 7. $a^6 - b^6$.

8. $y^5 - 5y^3 + 2y^2 + 6y - 4$. 9. $a^3 + \frac{17}{3}a^2b + \frac{1}{3}ab - 2ab^2 - \frac{1}{9}b^2$.

10. $(a^2 - b^2)x^{n+1}$. 11. $x^5 - a^5$. 12. $1 - x^6$.

13. $y^6 + 2y^4 - 7y^2 - 16$. 14. $\frac{1}{4}x^4 + y^4$.

15. $am + (an - bm)x + (ap - bn)x^2 - bpx^3$.

16. $a - (a^2 - b)x + cx^2 - (ac - b^2)x^3 + bcx^4$.

17. $x^8 - 3x^7 - 3x^6 + 6x^5 + 4x^4 - 5x^3 + 6x^2 - 12x + 6$.

18. $a^3 + b^3 - c^3 + 3abc$. 19. $x^3 + y^3 + 3xy - 1$.

20. $18x^8 + 27x^7 + 7x^6 + 3x^5 - 2x^4 + 65x^3 + 115x^2 + 49x + 6$.

21. $(x + y)^2 - (z + a)^2$.

22. $16a^2 + 24ab + 9b^2 - 4c^2 - 4cd - d^2$.

23. $16a^2 - 24ab + 9b^2 - 4c^2 + 4cd - d^2$.

24. $x^4 + 2x^3 + x^2 - y^4 + 2y^3 - y^2$. 25. $a^3 + 8b^3 - 27c^3 + 18abc$.

26. $81x^4 - 256a^4$. 27. $x^6 + 2x^3y^3 + y^6$.

28. $\frac{9}{8}x^4 - \frac{3}{2}ax^3 + \frac{1}{2}a^2x^2 - \frac{2}{9}a^4$. 29. $x^5 - a^5$.

30. $a^2x^{m+2} + abx^{m+2} + abx^{m+3} + b^2x^{n+3} - bx^3 - ax^2$.

31. $x^{2m} + x^{2m}y^m - x^my^m - y^{3m}$. 32. $x^8 - a^8b^8$. 33. $x^{16} - 1$.

34. $x^8 + x^4a^4 + a^8$. 35. $x^4 - y^4 - 4y^3 - 6y^2 - 4y - 1$.

EXERCISE XXIV [a]. (PAGE 53.)

1. $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$. 2. $x^6 - 3x^4 + 3x^2 - 1$.

3. $2x^5 - 18x^4 + 39x^3 - 25x^2 + x + 1$.

4. $4x^6 - 5x^5 + 8x^4 - 10x^3 - 8x^2 - 5x - 4$.

5. $21x^8 + 14x^7 - 49x^6 - 8x^5 - 10x^4 + 41x^3 - x^2 - 14x + 2$.

6. $3x^6 + 7x^5 - 12x^4 + 2x^3 - 3x^2 + 13x - 6$.

7. $4x^6 - 8x^5 + 4x^4 - 12x^3 + 12x^2 - 6x + 9$.

8. $x^8 - 3x^6 + 6x^4 - 7x^2 + 3$. 9. $x^6 - 57x^4 + 266x^2 - 1$.

10. $18x^8 + 21x^7 + 8x^6 + x^5 + 63x^3 + 96x^2 + 43x + 6$.

11. $x^6 - 3x^4a^2 + 3x^2a^4 - a^6$. 12. $1 - x^7$.

13. $4 - 12a + 5a^2 + 14a^3 - 11a^4 - 4a^5 + 4a^6$.

14. $1 + x + x^3 + x^4 + x^{17}$.

15. $akx^5 + (al + bk) x^4 + (am + bl + ck) x^3 + (an + bm + cl) x^2$
    $(bn + cm) x + cn.$

[b.]

2. 0.　　　　3. $y^4 - 7y^2 + 10.$　　　　5. $729x^6 - 117649.$

6. $2a^2 - 2ap - 2a^2n + p^2 + 2anp - 2an + np + 2an^2.$

7. 0, put $a = b + c.$

EXERCISE XXV.　(Page 56.)

1. $3x$; $7x$; $-3x^2$; $-5x^2.$　　　　2. $-a^3c^4$; a^4 ; $-7a^2b^2c^2.$

3. $3a^2$; $-2x$; $-2x^{m-3}.$　　　　4. $3a^5b^5c$; $\frac{7}{4}xy$; $-\frac{1}{2}x.$

5. $-a^2c$; $-a^{p-1}$; $ax^4.$　　6. $-2a^{m-1}b^{n-1}$; $-a$; ma ; $-2x^{2p+2}.$

7. $3a^{m-n}p^{n-1}$; $4a^3 (x - y)^{n-3}$; $-(a + b)^{n-m}.$

8. $-4mx^4 \div 5a^3$; $-3ab \div 4c$; $12a \div c.$

9. $a^2c^2 \div b$; $-a^{m-1}$; $x^{n-4} \div y^{n-4}.$　　10. $mx^m \div ny^n$; $a^2b^2c^2 \div x^3.$

EXERCISE XXVI.　(Page 57.)

1. $x - 2y$; $-x^2 + y^2$; $a^2b - a.$

2. $1 - 3ax - 4a^2x^2$; $-1 \div x + 2abx.$

3. $-a + b + c$; $-a + b + b^2$; $\frac{1}{4}xy - \frac{1}{6}.$

4. $-3mx^{m-n} + 2am^3 - \frac{2}{7}a^4mx^{p-n}.$

5. $(a + b)$; $4 (a - b)^2$; $a^{n-3} + a^{n-4}.$

6. $-\frac{4}{5}x^3y^3 + \frac{8}{3}x^2y - 2y$; $3x^2 - \frac{36}{25}y + 4.$

7. $-a^n - x$; $a^{2n} - a^nx^n + x^{2n}.$　8. $6a^2xy - 5ay^2 + 3a^2xy - 4ay^2.$

9. $-\frac{1}{2}x^4 \div y + \frac{1}{4}x^5 - \frac{3}{4}x^2y^2 + \frac{1}{4}x^2.$

10. $\dfrac{2}{5c^2} - \dfrac{3}{5a^2} + \dfrac{4}{5a^2}$; $4x^6 - x^2 + \dfrac{3}{2x^2}.$

11. $-\dfrac{x}{4y} + \dfrac{1}{4} - \dfrac{3}{4y}$; $\dfrac{a^3}{x^3} - \dfrac{a^2}{x^2} + \dfrac{a}{x} - 1.$

12. $\dfrac{3x^2}{2a^2} - \dfrac{5}{2a} + 3 + \dfrac{a}{2x^2}$; $(a - b)^{n-1}.$

13. $\frac{3}{2} (a + b)^3 - (a + b) + \frac{1}{2}.$　　　14. $a^{m-n} - (a - b)^{m-n}.$

15. $2 (x + y)^{m-2} (x - y)^{n-2} - (x + y)^{p-2} (x - y)^{q-2}.$

16. $(a + b)^{m-3} (a - b)^{n-3} - (a + b)^{n-3} (a - b)^{m-3}.$

EXERCISE XXVII. (PAGE 60.)

1. $x + 7$. 2. $a - 6$. 3. $3x - 2$. 4. $a - 24$.

5. $3x + 1$. 6. $3x - 7$. 7. $3x + 2$. 8. $4x + 3$.

9. $3x - 2y$. 10. $x - 7$. 11. $3x + 4$. 12. $5x - 1$.

13. $x^2 - y^2$. 14. $9x^2 + 4y^2$. 15. $8x + 3y$. 16. $x^2 + 14x$.

17. $4x^2 - x$. 18. $x^2 + 3x + 1$. 19. $a^2 + a - 1$.

20. $a^2x^2 + ax + 1$. 21. $2ab$. 22. $a^4 - 3ba^2 + 2b^2a$.

23. $x^2 + 2xy + 2y^2$. 24. $x^2 - 5x + 6$. 25. $x^2 - 2x + 2$; $-100x$.

26. $24x^2 - 2ax - 35a^2$. 27. $x^5 + xy^6 + 7ax$.

28. $-5a^2 + 4bd - 8cf$. 29. $3a^2 - 5b^2 + 3c^2$.

30. $3x^2 - x + 2$. 31. $a - b - c$. 32. $5a^2 + 3x^2$.

33. $p^2q + 4pq^2 + 2q^3$. 34. $x^2 - mx + m^2 - n$; $(m^2 - m^3)x$.

35. $x + y$; $y^{m+1} + 2xy^m$. 36. $x^{2n} - 2x^ny^n + y^{2n}$.

37. $ax^2 - bx^2 - a^2x + abx + a^3 - a^2b$. (Read a^3b^2 for a^2b^2 in text.)

EXERCISE XXVIII. (PAGE 61.)

1. $a^2 + b^2 + c^2 - ab + bc + ca$. 2. $x^2 - (a + b)x + ab$.

3. $y^4 - (m - 1)y^3 - (m - n - 1)y^2 - (m - 1)y + 1$.

4. $p^2 + q^2 + r^2 + pq + qr - rp$.

5. $1 - x + 2y + x^2 + 2xy + 4y^2$, $1 + x - 2y + x^2 + 2xy + 4y^2$.

6. $x^2 + y^2 + z^2 + 1$. 7. $3x^2 - x - 2$, rem $2x+1$. 8. $2x^2 - x + 1$.

EXERCISE XXIX. (PAGE 64.)

1. $2x^3 + 3x^2 + 4x + 7$. 2. $x^2 - 2x + 4$.

3. $5x^2 - 10x + 2$, $3x^2 - 10x + 1$. 4. $x^3 - 3x^2 + 3x - 1$.

5. $4x^3 - 3x^2 + 2x + 2$. 6. $5x^2 - 12x + 12$, $12x - 72$.

7. $5x^2 + 10x + 5$, $-5x^2 - 10x + 27$. 8. $10x^3$, $10x^4 - 100$.

9. $1 - 2x + 3x^2 - 4x^3 + 5x^4$; $a^4 + 2a^3 + 3a^2 + 4a + 5$.

10. $x^2 + 2xy + 3y^2$; $m^2 - 2m + 3$.

11. $x^4 + 2x^3 + 3x^2 + 2x + 1$; $a^4 - 2a^3b + 3a^2b^2 - 2ab^3 + b^4$.

12. $3x^4 - 2x^3 - 2x + 3$.

13. $x^4 - 3x^2 - 4x + 15$, $54x^2 - 56x + 27$.

14. $x^5 - 3x^4 - 2x^3 + 2x^2 + 3x - 1$, $5x$.

15. $2x^3 - x^2 - 2x + 4$, $24x^2 - 12x + 10$.

16. $x^4 + 4x^3 + 6x^2 + 9x - 4$, rem. 5. 17. $2x^2 - 4x + 3$.

18. $x^8 - 2x^6 + 3x^4 - 2x^2 + 1$: $x^4 - x^3 + x^2 - x + 1$.

19. $x^2 + (a + b) x + ab$; $x^2 - ax + bx - ab$.   20. $2y^2 - cy - \frac{1}{2}$.

21. $5x^4 - 24x^3 + 99x^2 - 400x + 1601$. rem. $- 6400$.

22. $x^5 - x^4 - x^3 + x^2 + 2x + 1$, rem. 101.

23. $2x^4 - x^3 + 2x^2 - 3x + 1$, rem. 10.

24. $\frac{1}{3}x^5 + \frac{4}{3}x^4 - x^3 + \frac{1}{3}x^2 + \frac{2}{3}$, rem. $- 3x^3 + 21x^2 - 3x + 14$; take factor 3 out of divisor and divide resulting quotient by 3.

EXERCISE XXX. (Page 69.)

1. 8.   2. 1.   3. $4w - 18$.   4. 3.   5. 2.   6. 1.

7. 2.   8. 4.   9. 12.   10. $(a^2 + ab + b^2) \div 2 (a + 2)$.

11. $(n - b + a^2) \div (a - m + 2ab + c)$.

12. $(bc - ab) \div \{a + c + b^2 + (a - c)^2\}$.

13. $(a^2 + b^2 + ab) \div (a + b)$.   14. $6c \div (30 - 11a + 3b + 2c)$.

15. $x = \frac{1}{2} (a + b)$; write P for $x - a$, and Q for $x - b$, and equation becomes $P^3 \div Q^3 = \{P - (a - b)\} \div \{Q + (a - b)\}$, and on clearing of fractions $P^2 - Q^2$ will prove to be a *factor* ; $\therefore P^2 - Q^2 = 0$, $P + Q = 0$, etc. Or, multiply out.

16. $(c^2 - ab) \div (a + b - 2c)$;
    equation is $(x + a) \div (x + b) = (2x + a + c) \div (2x + b + c)$; complete the divisions, square and transpose ;
    $\therefore (c - b) \div (x + b) = (a - b) \div (2x + b + c)$, etc.

17. 19.   18. 9.   19. 9.

20. 72 ; remove brackets and combine numerical quantities.

21. $4\frac{7}{9}$.   22. 4.

23. 3 ; equation is $\frac{7}{13}x + \frac{7}{18}x + \frac{3}{4}x - \frac{10}{11}x = \frac{8}{13} + \frac{2}{7} + \frac{3}{11} + \frac{1}{8}$; or $\frac{215}{234}x - \frac{37}{77}x = 8051 \div 66 \times 91$;
    i. e. $8051x \div 77 \times 234 = 8051 \div 66 \times 91$, etc.

24. $(2ab^2 - 5a) \div (2a - 2b + 3)$.   25. 8.

EXERCISE XXXI. (Page 72.)

1. 240.   2. 12 miles.   3. $8\frac{4}{7}$ miles.   4. 8 men.

5. $na + nb \div a$.   6. $mn (a - b) \div (mn - m - n)$.

7. Price $= \$(22x - 21z) y \div 20x (x - z)$.   8. 50 gal.

9. One-third.   10. 188 oz. ; $\frac{1}{20} (x + 32) = \frac{1}{12} (x - 56)$, where $x =$ wt. of lump.

11. 30 eggs.   12. 4.   13. 40.   14. $78\frac{5}{9}$.

15. 16200, 23000.   16. 63.   17. 30 gal.   18. $1080 \div 251$ miles.

19. B in $ac \div (a - b)$ days, A in $ac \div (c - a + b)$.

20. 22 gals., 9s.   21. $3.   22. 37, 38, 39.   23. 6, 9, 18.   24. 235.

25. Let $x =$ increase of rate, then $c \div a + x = c \div a - b$,
    $x = a^2 b \div (c - ab)$.

26. 7, 8, 9.   27. $2pqr \div (pq + qr + rp)$.

28. $(ma - b) \div (m - 1)$, $m (b - a) \div (m - 1)$.

29. 432.   30. $n (m - p) \div p$.

31. $20\frac{2}{9}$, $24\frac{2}{9}$, $11\frac{1}{9}$, $44\frac{4}{9}$; if $x =$ 1st part, $x + 4 =$ 2d, $\frac{1}{2}(x + 2) =$ 3d,
    $2(x + 2) =$ 4th, and their sum is 100.

EXERCISE XXXII [b]. (PAGE 78.)

1. $169a^2 - 52a + 4a^2$; $225x^2 - 15ax + \frac{1}{4}a^2$;
   $441x^2y^2 + 126x^2y + 9x^2$; $144a^2b^4 - 144a^3b^3c + 36a^4b^2c^2$.

2. $\frac{4}{9}x^4 + x^2y^2 + \frac{9}{16}y^4$; $18\frac{1}{16}a^4b^2 - 2a^3b^3 + \frac{16}{289}a^2b^4$;
   $289x^4y^6z^n - 2x^6y^6z^6 + \frac{1}{289}x^8y^6z^4$.

3. $1,024,144$; $1,096,004$; 12321; 5625; 2401.

4. $25a^{116}b^{14} + 60a^{149}b^{12} + 36a^{182}b^{10}$; $a^{224} - 2a^{112}b^{20} + b^{40}$;
   $5929a^{154} + 13552a^{77}b^{88} + 7744b^{176}$.

5. $169x^2 - 4$; $\frac{1}{4}x^2 - \frac{1}{400}$; $4x^4 - \frac{1}{144}y^4$.

6. $49x^{14} - 256x^2y^2$; $\frac{1}{49}a^{14} - \frac{1}{36}x^2y^2$; $x^{154} - y^{176}$.

7. $999,856$; 9879; 4875; 2499.

8. $x^{224} - b^{400}$; $25a^{116}b^{144} - 36a^{182}b^{114}$; $5929a^{154} - 7744b^{176}$.

9. $4x^2$.   10. $a^2 + b^2 + 2ab - c^2$; $x^2 - 2xy + y^2 - z^2$.

11. $4a^2 - b^2 + 6bc - 9c^2$; $y^2 - 4x^2 + 12xz - 9z^2$.

12. $(w + y)^2 - (x + z)^2$; $(s + t)^2 - (u + v)^2$.

13. $(a + d)^2 - (2b - 3c)^2$; $(3y + z)^2 - (x - 2k)^2$.

14. $2m^2 + 6ms - 8p^2 + 4pk - 12ps - 2mk$.

EXERCISE XXXIII [a]. (PAGE 80.)

6. $1 + 2x + 3x^2 + 2x^3 + x^4$; $1 - 2x + 3x^2 - 2x^3 + x^4$;
   $1 + 4x + 6x^2 + 4x^3 + x^4$; $1 - 4x + 6x^2 - 4x^3 + x^4$.

7. $16 + x^2 + 4y^2 + 8x - 16y - 4xy$; $25 + y^2 + 9z^2 - 10y - 30z + 6yz$;
   $1 - 2x - x^2 + 2x^3 + x^4$; $x^4 + y^4 + z^4 + 2x^2y^2 + 2y^2z^2 + 2z^2x^2$.

3. $1 + 2x^2 + 6x^3 + x^4 + 6x^5 + 9x^6$;
$1 - 2x^2 + 6x^3 + x^4 - 6x^5 + 9x^6$;
$4 - 4y + 9y^2 - 4y^3 + 4y^4$; $4x^4 + y^2 + 1 + 4x^2y - 2y - 4x^2$.

9. $1 - 2x + 5x^2 - 4x^3 + 4x^4$; $1 + 2x - 5x^2 - 6x^3 + 9x^4$;
$4a^4 - 7a^2 + 4 - 4a^3 + 4a$; $1 + 2a^2 + 2a^3 + 2a^5 + a^9$.

10. $1 + x^2 + b^2y^2 + 2x + 2by + 2bxy$;
$1 + a^2x^2 + b^2y^2 + 2ax + 2by + 2abxy$;
$1 + a^2x^2 + b^2y^2 - 2ax - 2by + 2abxy$;
$1 - 2ax^2 + 2bx^3 + a^2x^4 - 2abx^5 + b^2x^6$.

11. $1 + 2x + 3x^2 + 4x^3 + 3x^4 + 2x^5 + x^6$;
$1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$;
$1 - 2x - x^2 + 3x^4 + 2x^5 + x^6$.

12. $1 - 4ax + 10a^2x^2 - 12a^3x^3 + 9a^4x^4$;
$x^6 - 6x^5 + 13x^4 - 14x^3 + 10x^2 - 4x + 1$;
$x^6 - 4x^5 + 10x^4 - 4x^3 - 7x^2 + 24x + 16$.

13. $4a^2 + b^2 + 4c^2 - 4ab + 8ac - 4bc$;
$a^2 + \frac{1}{4}b^2 + \frac{1}{4}c^2 - ab + ac - \frac{1}{2}bc$;
$\frac{1}{4}a^2 + \frac{1}{4}b^2 + c^2 - \frac{1}{2}ab - bc + ac$;
$\frac{1}{4}a^2 + b^2 + \frac{1}{9}c^2 - ab - \frac{2}{3}bc + \frac{1}{3}ac$.

[b.]

1. $(2x + y)^2 \div 160$. 2. $(a + b - c)^2 \div 100$. 3. $(x + 2y + 3w + 4z)^2$.

4. $6(ab + bc - ca)^2$. 6. $4(w^2 + x^2 + y^2 + z^2)$. 7. $8x^3y$.

8. $a^8 + 2a^6 + 3a^4 + 2a^2 + 1$.

10. $2a^2b^2 + 2b^2c^2 + 2c^2a^2 - a^4 - b^4 - c^4$; see Ex. 3, p. 128.

EXERCISE XXXIV [b]. (PAGE 84.)

4. $(3x + 4y)^2 - 25z^2$; $(2a + 4c)^2 - 9b^2$.

5. $\{(x^3 + 2x^2 + 4) - 3x\} \times \{(x^3 + 2x^2 + 4) - 5x\}$
$= x^6 + 4x^5 - 4x^4 - 8x^3 + 31x^2 - 32x + 16$;
$\{(x + z + w) + y\} \{(x + z + w) + 3y\}$
$= (x + z + w)^2 + 4y(x + z + w) + 3y^2$.

6. $x^3 + 9x^2 + 26x + 24$; $x^3 + 14x^2 + 55x + 42$;
$x^3 + 9x^2 + 23x + 15$.

7. $x^3 - 9x^2 + 26x - 24$; $x^3 - 14x^2 + 55x - 42$;
$x^3 - 9x^2 + 23x - 15$.

8. $x^3 + 3x^2 - 10x - 24$; $x^3 - 12c^2 + 29x + 42$;
$x^3 + x^2 - 17x + 15$.

9. $8x^3 + 12x^2 + 22x + 6$; $8x^3 - 12x^2 + 22x - 6$;
     $8x^3 - 4x^2 - 10x + 6.$

10. $x^4 + x^3(y + z + w + k) + x^2(wy + wz + wk + yz + yk + zk)$
     $+ x(yzw + yzk + zwk + ykw) + yzwk$;
     $x^4 - (a + b + c + d)x^3 + (ab + ac + ad + bc + bd + cd)x^2$
     $- (abc + abd + acd + bcd)x + abcd.$

11. $w^3 + 3w^2r + 3wr^2 + r^3$; $w^4 + 4w^3r + 6w^2r^2 + 4wr^3 + r^4$;
     $8w^3 + 12w^2r + 6wr^2 + r^3$; $w^4 + 8w^3r + 24w^2r^2 + 32wr^3 + 16r^4$;
     $w^3 - 3w^2r + 3wr^2 - r^3$; $w^4 - 4w^3r + 6w^2r^2 - 4wr^3 + r^4.$

12. $k^5 + 15k^4s + 90k^3s^2 + 270k^2s^3 + 405ks^4 + 243s^5$;
     $a^6 - 12a^5b + 60a^4b^2 - 160a^3b^3 + 240a^2b^4 - 192ab^5 + 64b^6$;
     $8a^3 - 6a^2w + \frac{3}{2}aw^2 - \frac{1}{8}w^3$; $\frac{1}{8}a^3 + \frac{3}{2}a^2w + 6aw^2 + 8w^3$;
     $27a^6 - 9a^4 + a^2 - \frac{1}{27}.$

13. $1320a^7b^3$; $- 22680a^4b^3$; $- 2a^7x^3.$

14. $1485a^2b^{53} + 55ab^{54} + b^{55}$; $2145x^2y^{64} + 66xy^{65} + y^{66}$;
     $- 6655a^2 + 121a - 1.$

15. $54a^2b^2$; $540a^3b^3$; $1680a^4.$ 16. 1.21662924 ; $1.7101875.$

EXERCISE XXXV [a].   (PAGE 87.)

1. $x^4 + 2x^3 - 85x^2 - 86x + 1680.$

2.   Write k for $x + a$, m for $x + b$, $\therefore$ product $= k^4 + k^2m^2 + m^4$
     $= 3x^4 + 6x^3(a+b) + x^2(7a^2 + 4ab + 7b^2)$
     $+ x(4a^3 + 2a^2b + 2ab^2 + 4b^3) + (a^4 + a^2b^2 + b^4).$

3. $a^2b^2 + c^2d^2 - a^2c^2 - b^2d^2.$ 4. $a^3 + b^3 + c^3 - 3abc.$

5. $x^5 - px^4 + qx^3 - qx^2 + px - 1.$

6. $a^3(x^3 - 1) - a^2(x^3 + x^2 - 2) + a(4x^2 + 3x + 2) - 3(x + 1).$

7. $w^2 - z^2.$ 8. $8x^3.$ 9. $24xyz.$ 10. $zw + xy.$

11. $6xyz.$ 12. $4x^2y^2.$ 13.   See p. 85, H, (3).

EXERCISE XXXVI [b].   (PAGE 88.)

7. $a + b - c$; $x - 2y - 3z.$ 8. $a^2 - 2ab + b^2$; $a^2 + ab + b^2.$

9. $1 + 2x + 3x^2$; $3a^2 + 2a + 3.$ 12. $x + 4$; $2x - 3b.$

13. $a + 8b$; $2a - 7b.$ 14. $1 + a^2.$ 15. $x^2 - 2x + 1.$

[c.]

1. $\dfrac{3a}{5} - \dfrac{5}{3a}$; $\dfrac{a}{2b} - 2$; $\dfrac{8m}{3w} + 2.$

2. $\frac{1}{8}x^2 + \frac{1}{2}x - 1$; $x^2 + x - \frac{1}{2}$; $x^2 + 2x + 1$.

3. $2x + 3y - 5z$; $2x^2 - x + 1$.   4. $\frac{4y}{x} - 4 + \frac{x}{y}$; $x - 2 - \frac{1}{x}$.

5. $\frac{3a}{b} - \frac{1}{3} + \frac{2b}{3a}$.   6. $4a - 3b$; $\frac{2}{x^2} - 3x$.

7. $1 - 2x + 3x^2$; $\frac{1}{2}x - 1$.   8. $a + 2b - c$.   9. $\frac{x}{3} - 1 + \frac{3}{x}$; $\frac{x}{3} + 2$.

10. Cube both sides by formula G (2), p. 85.

EXERCISE XXXVII.

1. $a^2b + b^2a$; $a(a+b)^2 + b(b+a)^2$;
$ab(b-c) + bc(c-a) + ca(a-b)$; $a^2bc + b^2ca + c^2ab$;
$a(b+c) + b(c+a) + c(a+b)$.

2. $(a-b)(b-c) + (b-c)(c-a) + (c-a)(a-b)$;
$a^2(b-c) + b^2(c-a) + c^2(a-b)$;
$a(b-c)^2 + b(c-a)^2 + c(a-b)^2$;
$(x-a)(b-c)^2 + (x-b)(c-a)^2 + (x-c)(a-b)^2$.

3. $a^3 + b^3 + c^3 + d^3$;
$a^2(bc + bd + cd) + b^2(ac + ad + cd)$
$+ c^2(ab + ad + bc) + d^2(ab + bc + ac)$;
$a^2(b+c+d) + b^2(c+d+a) + c^2(d+a+b) + d^2(a+b+c)$;
$a + b + a + c + a + d + b + c + b + d + c + d$;
$ab + ac + ad + bc + bd + cd$;
$a^2(a-b) + b^2(b-c) + c^2(c-d) + d^2(d-a)$;
$(a-b)^3 + (a-c)^3 + (a-d)^3 + (b-c)^3 + (b-d)^3 + (c-d)^3$.

4. $(a-b)^2(b-c)^2 + (a-c)^2(c-d)^2 + (a-b)^2(b-d)^2 + (b-c)^2(c-d)^2$;
$(x-a)(b-c)^2 + (x-b)(c-d)^2 + (x-c)(d-a)^2 + (x-d)(a-b)^2$.

13. $a, b, -c$; $a, -b, c$; $a, -b, -c$.   14. a, b, c ; a, b.

15. a, b, c ; a, b ; $a, -b$.   16. a, b, c ; ax and by, x and y.

17. a and b ; a, b, c.   18. a, b, c.   19. a, b ; a, b.

20. a, b, c.   21. a^2b.   22. a^4, a^3b, a^2b^2, abc^2.

23. x^3, x^2y, xyz.   24. a^3b.   25. ab^3, ab^2c.

26. xy^2, xyz.   27. x^3, x^2y ; x^4, x^3y, x^2y^2 ; x^5, x^4y, y^3y^2.

28. x^3, x^2y, xyz.   29. a^4, a^3b, a^2b^2, a^2bc ; $x^5, x^4y, x^3yz, x^2y^2z, x^3z^2$.

30. a^3, a^2b, abc ; $x^7, x^6y, x^5y^2, x^4y^3$.

31. $a^3 + b^3 + c^3 + d^3 - 3(abc + abd + bcd + cda)$.

32. $a + b - c$; $a - b + c$; $-a + b + c$; $a - b - c$.

33. $\frac{1}{2} \{(a - b)^2 + (b + c)^2 + (c + a)^2\}$;

   $\frac{1}{2} \{(a + b)^2 + (b + c)^2 + (c - a)^2\}$;

   $\frac{1}{2} \{(a + b)^2 + (b - c)^2 + (c + a)^2\}$;

   $\frac{1}{2} \{(a + b)^2 + (b - c)^2 + (c + a)^2\}$;

   the three expressions are derived from the *first* by, respectively, substituting $-c$ for c, $-b$ for b, $-a$ for a; observe, also, that $(-a - c)^2 = + (a + c)^2$.

EXERCISE XXXVIII.   (Page 96.)

1. $3 (a^2 + b^2 + c^2) - 2 (ab + \&c.)$.   2. 0.   3. $2 (xy + yz + zx)$.

4. $6 (a^2 + b^2 + c^2) - 2 (ab + bc + ca)$.

5. $2 (x^2 + y^2 + z^2 - yx - yz - zx)$.

6. $14 (a^2 + b^2 + c^2) - 14 (ab + bc + ca)$.

7. $4 (a^2 + b^2 + c^2 + d^2)$.   8. $4 (a^2x^2 + b^2y^2 + c^2z^2)$.

9. $2 (a^3 + b^3 + c^3) + 6 (a^2b + \text{etc.}) - 12abc$.

10. $a^2 + b^2 + c^2 + d^2$.   11. $3 (a^2 + b^2 + c^2 + d^2) + 2 (ab + \text{etc.})$.

12. 0.   13. $6abc$.   14. $abc (a + b + c)$.

15. $4 (x^4 + y^4 + z^4) + 24 (a^2b^2 + b^2c^2 + c^2a^2)$.

16. NOTE.—The first term in each of the binomial *factors* should have *index* 2; *i. e.*, a^2 for a, etc. Multiply out, or use identity, $x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$.

17. Multiply out and subs. for s.

18. $rs = (a + b)^2 - (c - d)^2$, the other pairs by symmetry; result is $4 (ab + ac + ad + bc + bd + cd)$.

20. Type terms are a^4, $2a^3 (b + c)$, a^2b^2, and both expressions reduce to same form. *Or*, use identity, Ex. 7, p. 105, putting $a - b$ for a, $b - c$ for b, and $\therefore$ $a - c$ for $a + b$.

EXERCISE XL.   (Page 98.)

1. $(a - b) (x + 2y)$.

2. $(a + b) (2x - 3y)$.

3. $(a + x) (a - b)$.

4. $(c - d) (ab - c)$.

5. $(m + n) (x^2 - a)$.

6. $(a + b) (a - c)$.

7. $(a + b) (3x + y)$.

8. $(a - bc) (1 - x)$.

9. $(a - b) (c + y)$.

10. $(a + x) (a + b)$.

11. $(3a - b) (x - y)$.

12. $(7 - x) (a - bc)$.

13. $(r - s)(3p + q)$.   14. $(1 - a)(1 - b)$.

15. $(3x - a)(2x + y)$.   16. $(a^2 - 1)(a + 1)$.

17. $(x - 1)(3bx - 1)$.   18. $(xy - z)(a + bc)$.

19. $(a - 1)(a^2 + 1)$.   20. $(x + f)(2a + b)$.

21. $(x^2 + a^2)(a - 3c)$.   22. $(x - y)(x - 3)$.

23. $(x^2 - 1)(2a^2 - 1)$.   24. $(b - 1)(c - 1)$.

25. $(a^2x^2 - c)(a^2x^2 - b)$.   26. $(3b^2 - 1)(1 - 3a^2)$.

27. $(x^2 - a^2)(x^2 + ax + a^2)$.   28. $(a - b)(x - y + z)$.

29. $(a + b)(ax + by + c)$.   30. $(ax^n - b)(bx^n + a)$.

31. $(a - 1)(a + b)$.   32. $(2 + x^n)(3 - y^n)$.

33. $(1 - b)(a - b + c)$.   34. $(a - x)(2pq - 3bf)$.

35. $(1 - x^2)(1 + x^2 + p + q)$.   36. $(2p^n - 3q^n)(r^n - 2s^n)$.

37. $(a + b - c)(d - e + f)$.   38. $(1 + p + q)(1 - a + b)$.

EXERCISE XLI. (PAGE 100.)

11. $\left(\dfrac{a}{b} - \dfrac{b}{a}\right)^2$; $(1 - x^m)^2$; $\{(2x - 3y) - (2x + 3y)\}^2$.

12. $\{(a^2 + ab + b^2) - (a^2 - ab + b^2)\}^2$; $\left\{\left(\dfrac{x}{y}\right)^m - \left(\dfrac{y}{x}\right)^m\right\}^2$.

13. $(x + y + z)^2$; $(p - q + r)^2$.

14. $(a - 2b + 3c)^2$; $(1 - x + y)^2$.

15. $(3a + 2b + c)^2$; $(2a^2 - 3a + 4)^2$.   16. $(3ax + 2by + cz)^2$.

17. $(2a^2 - 3b + 4c)^2$; $(a^2 - b^2 - c^2)^2$.

18. $\pm 4xy$; $\pm xy$; x^2y^2; $-10xy$; $\pm 4x^2y^2$.

19. $\pm 6ay$; $\pm 10a^3b$; $\pm 12x^3y^2$; $\pm 2a^nb^n$; a^2.

20. a^4; z^2; $\frac{1}{4}$; $\frac{1}{4}$; 4; b^2.   21. $\frac{1}{4}b^2$; $\pm 4bxy^2$; ± 2; ± 2; x^4.

22. 7; $\frac{1}{4}$; $b^2 \div 4a^2$; $25 \div 4$; $49 \div 4$.

23. $81 \div 16$; $x^2 + 4$; $x^2 + 13$; $-c + \frac{1}{4}b^2$.

EXERCISE XLII [b]. (PAGE 103.)

NOTE.—The two factors in each case are expressed with the double sign $\pm$.

1. $a + b \pm c$; $2(x + y) \pm z$; $x \pm (y + z)$; $2 \pm (a + b)$.

2. $p + 2q \pm r$; $4x \pm (a + 3b)$; $2m \pm (p - q)$; $2x(-4y)$.

3. $1 \pm (b - c)$; $a + b + c \pm x$; $(8 + x)(10 - x)$; $b - c \pm (a - x)$.

4. $3\{2(a^2 - bc) \pm (b^2 - ac)\}$; $a - 5b \pm 1$; $1 \pm (x - y + z)$;
$(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$; $(a - 3c)(a + 4b + 3c)$.

5. $(-a + b - 4c)(3a - 5b + 4c)$;
$(1 - a + b)(1 - a - b)(1 + 2a - a^2 + b^2)$;
$(12x - 1)(2x + 7)$.

6. $(x - z \pm y)(x + z \pm y)$; $4(x + z)(y + u)$;
$\{x \pm (y + z)\}\{x \pm (y - z)\}$.

7. $(x - z) \pm (y - u)$; $a \pm (x - y)$; $x \pm (y + z)$.

8. $x \pm (y - z)$; $x \pm (y + z)$; $x + z \pm y$; $x^2 \pm (x - 1)$.

9. $(x + a) \pm (y + z)$; $(a - c) \pm (b - d)$;
$(a^8 + b^8)(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$;
$(a^2 + 6a + 5)(a^2 + 2a + 3)$.

10. $a - b \pm (x - y)$; $a^2 + a \pm (b^2 - b)$.

11. $(x + b)(a \pm x)$; $\{a - d \pm (b - c)\}$; $ab \pm c(a - b)$.

12. $\{c \pm (a - b)\}\{a + b \pm c\}$; $x^2 + y^2 \pm (z^2 + 1)$; $a - d \pm (b - c)$.

13. $2a \pm (b - 3c)$; $b \pm (2a - 3c)$; $2a \pm (b + 3c)$.

14. $3c \pm (2a - b)$; $(a + c) \pm (b + d)$; $(a + d) \pm (b + c)$.

15. $(b + c) \pm (a + d)$; $(a + d) \pm (2b - 3c)$.

16. $3c + d \pm (a - 2b)$; $(a - 3c) \pm (2b - d)$.

17. $\{a + d \pm (b - c)\}\{b + c \pm (a - d)\}$.

18. $(x^2 + 1 \div y^2)(x + 1 \div y)(x - 1 \div y)$; $x^4 \pm \frac{1}{16}$, etc.;
$x^3(x^4 - 25) - \frac{1}{4}(x^4 - 25) = (x^4 - 25)(x^3 - \frac{1}{4})$, etc.;
$(x^4 - 16)(x^3 + 1)$, etc.

EXERCISE XLIII. (PAGE 105.)

1. $3x^2 + y^2 \pm xy$.
2. $4a^2 - b^2 \pm 3ab$.

3. $3a^2 + b^2 \pm 5ab$.
4. $5m^2 + 4n^2 \pm 7mn$.

5. $x^2 + 1 \pm x$.
6. $x^2 + 4 \pm 2x$.

7. $x^2 + 25 \pm 5x$.
8. $x^2 + \frac{9}{4} \pm \frac{3}{2}a$.

9. $x^2 - y^2 \pm 3xy$.
10. $x^2 + \frac{16}{9} \pm \frac{4}{3}x$.

11. $a^2 - y^2 \pm 2ay$.
12. $m^4 - n^4 \pm 4mn$.

13. $9a^2 + b^2 \pm 3ab$.
14. $4a^2 + b^2 \pm 6ab$.

15. $5p^2 - 4q^2 \pm pq$.
16. $9x^2 - y^2 \pm 4xy$.

17. $2x^2 - 1 \pm 2x$.
18. $\frac{1}{2}x^2 + y^2 \pm xy$.

19. $x^2 + 2a^2y^2 \pm 2axy$.
20. $2a^2 + y^2 \pm \frac{7}{2}ay$.

21. $x^4 + y^4 \pm x^2 y^2$, etc.: $x^4 + 1 \pm x^2$.

22. $a^2 x^4 + 1 \pm a x^2$; $x^4 + 2y^4 \pm 2xy$.

23. $(a + b)^2 + c^2 \pm 3c(a + b)$; $1 + 2c^2 \pm 2x$.

24. $4x^2 + 2(y - z)^2 \pm 5x(y - z)$; $1 + 5z^4 \pm 3z^2$.

25. $1 + 2a^4 \pm 2a^2$; $a^2 + 9b^2 \pm 9ab$.

26. $2(1 + a + a^2)^2$; $x^2 + 1 \div y^2 \pm x \div y$.

27. $x^2 + 1 \div 2y^2 \pm x \div y$; $a^2 + 2 \div a^2 \pm 2$;
$(a + b)^2 + (a - b)^2 \pm (a^2 - b^2)$.

28. $c^2 + 2(a + b)^2 \pm 2c(a + b)$; $1 \div a^2 + 1 \div b^2 \pm 1 \div ab$;
$3 \div a^2 + 1 \div b^2 \pm 3 \div ab$.

EXERCISE XLIV [a]. (PAGE 108.)

17. $(m^3 + 21)(m^3 + 19)$. 18. $(a^2 x + 39)(a^2 x + 1)$.

19. $(x^n + 7)(x^n + 12)$. 20. $(x + 17)(x + 23)$.

21. $(x^n + 12)(x^n + 4)$. 22. $(x + 33)(x + 27)$.

23. $(a + 27)(a + 13)$. 24. $(a + 18b)^2$.

25. $(a^2 x + 81)^2$. 26. $(x - 4)^2$. 27. $(x - 15)^2$.

28. $(x - 19)^2$. 29. $(x - 20)^2$. 30. $(x - 50)^2$.

31. $(x^3 - 5)(x^3 - 25)$. 32. $(m - 17n)(m - 5n)$.

33. $(x - 13y)^2$. 34. $(x^2 - 5y^2)(x^2 - 4y^2)$.

35. $(a - 27b)(a - 2b)$. 36. $(4 - x)(3 - x)$.

37. $(26 - ab)(5 - ab)$. 38. $(a - 25)(a - 15)$.

39. $x^2 + 1 \pm \frac{10}{3}x$. 40. $(x^3 - 27)(x^3 - 8)$.

41. $3x(x - 2)(x - 8)$. 42. $a(x - 5)(x - 6)$.

43. $x^2 + 60 \pm 17x$. 44. $(x^n - 7)(x^n - 37)$.

45. $(a + b - 4)(a + b - 3)$. 46. $(13 - ax)(11 - ax)$.

47. $(1 - 8x^2 y^2)(1 - 51x^2 y^2)$. 48. $(a - 27b)^2$.

49. $x^2(a - 15bx)(a - 5bx)$. 50. $(m - 19)^2$.

51. $(p - 27q)^2$. 52. $\{(x - y)^n - 33\}\{(x - y)^n - 11\}$.

[b.] (PAGE 109.)

1. $(a^2 + 1)(a^2 - 2)$. 2. $(a + 3)(a - 2)$.

3. $(x - 3)(x + 2)$. 4. $(x - 16)(x + 3)$.

5. $(x + 12)(x - 7)$. 6. $(y + 12)(y - 5)$.

7. $(a + 20)(a - 7)$.

8. $(a + 25b)(a - 12b)$.

9. $(x + 12)(x - 11)$.

10. $(x - 10)(x + 2)$.

11. $(y^2 - 10a^2)(y^2 + 5a^2)$.

12. $(ab - 4)(ab + 1)$.

13. $3(az^2 - 14)(az^2 + 1)$.

14. $(a^4 - 20)(a^4 + 5)$.

15. $(abc + 11)(abc - 2)$.

16. $(a^2b^2 - 30)(a^2b^2 + 3)$.

17. $(x^2 - 48)(x^2 + 8)$.

18. $(x^n - 16)(x^n + 3)$.

19. $(x + y - 19)(x + y + 18)$.

20. $\{a - 30(b+c)\}\{a + 12(b+c)\}$.

21. $(x^{2n} + 4)(x^{2n} - 3)$.

22. $(20 + a)(19 - a)$.

23. $(13 - ab)(5 + ab)$.

24. $(12 - m)(17 + m)$.

25. $3y(a + 14bx)(a - 2bx)$.

26. $(2x + 7)(2x + 5)$.

27. $(3x + 7)(3x + 5)$.

28. $(2x^2y - 7z^2)(2x^2y + 6z^2)$.

29. $(7a - 8b)^2$.

30. $x(2b - y)(2b - 5y)$.

31. $x^4(8y^3 - 10z)^2$. 32. $(a^2 - 40b^2)(a^2 + 5b^2)$. 33. $(11x^2 - 13y)^2$.

34. $3(x^2 + y^2)(3x^2 - 4y^2)$; where $x = a - b$ and $y = c$.

35. $(8x^n - 2b^n)(8x^n + b^n)$.

36. $(\frac{1}{2}x^2 + 7)(\frac{1}{2}x^2 - 6)$.

37. $(\frac{3}{4}a + 7b)(\frac{3}{4}a - 8b)$.

38. $\left(\frac{3}{x} - 7\right)\left(\frac{3}{x} + 9\right)$.

39. $\left(\frac{4}{x} + 19\right)\left(\frac{4}{x} - 20\right)$.

40. $(5x^2 + 21)(5x^2 - 31)$.

EXERCISE XLV. (PAGE 112.)

1. $(2x + 1)(2x + 3)$.

2. $(4x + 1)(x + 3)$.

3. $(3x + 3)(5x + 4)$.

4. $(3x + 2)(2x + 1)$.

5. $(2x + 5)(3x + 4)$.

6. $(2x + 7y)(4x + 3y)$.

7. $(4a + 9)(a + 1)$.

8. $(1 + m)(7 + 3m)$.

9. $(x + 5)(4x + 3)$.

10. $(x + 7)(3x + 2)$.

11. $(4x - 3)(3x + 2)$.

12. $(4x + 3)(3x - 2)$.

13. $(4x + 7)(3x - 5)$.

14. $(4x - 7)(3x + 5)$.

15. $(3x + 2)(2x - 1)$.

16. $(5x - 1)(2x - 3)$.

17. $(3x + 4)(5x - 2)$.

18. $(x - 7)(7x - 1)$.

19. $(5x + 2y)(3x - 5y)$.

20. $(a^2 - 19)(a^2 + 17)$.

21. $(3m + 20)(2m - 19)$.

22. $(2a + 20)(3a - 19)$.

23. $(3x + 7y)(4x - 5y)$.

24. $(3 - 12x)(5 + 11x)$.

25. $(5x^2 - 1)(4x^2 + 1)$.

26. $(15a - 1)(a + 15)$.

27. $(12x - 7)(2x + 3)$.

28. $(6 - y)(3 - 5y)$.

29. $(8a + b)(3a - 4b)$.

30. $(8 - 9y)(3 + 8y)$.

31. $(28x^2 - 25)(x^2 + 5).$ 32. $4(14x + 5y)(x - y).$

33. $4(7x - 5y)(2x - y).$ 34. $4(14x - 5y)(x + y).$

35. $(8a - 5b)(7a - 4b).$ 36. $2(28y + 1)(y - 10).$

37. $(8y + 5z)(9y - 8z).$ 38. $(9y + 3a)(4y - 5a).$

39. $(56a^2 + 4b^2)(a^2 - 5b^2).$ 40. $(56a - 5b)(a - 4b).$

41. $(13x + 12y)(3x - 4y).$ 42. $(3x + y)(13x - 11y).$

43. $(39x - 26)(x + 1).$ 44. $(12x + 13y)(5x - 8y).$

45. $(1 - 13x^2)(1 + 11x^2).$ 46. $(a^n - 13b^n)(a^n + 11b^n).$

47. $(3x^3 - 21)(4x^3 + 11).$ 48. $(17x - 1)(x + 17).$

EXERCISE XLVI. (Page 114)

1. $(x - a)(x^2 - 2x - 1).$ 2. $(x - a)(x^2 - px + q).$

3. $(my - n)(ay^2 + by - c).$ 4. $(2b - c)(x^2 - 2bx + b).$

5. $(nx - a)(x^2 - x - 1).$

6. $(bx - a)\{(m + 1)b^2x^2 + (m + 1)(n + 1)abx + (n + 1)a^2\}$;
   multiply out, take m-terms for one group, etc.

7. $(y - b)(y - a)^2.$ 8. $(x - b)(x - a)(x + 2b).$

9. $(x+p+q)(x+q-p)(x-2q).$ 10. $(x - a)(x + b)(x + 3).$

11. $(x+b)\{x(x-1)-a(x+1)\}.$ 12. $(2x - a)(2x^2 + 4x - 3).$

13. $(py + q)(y^2 - y + 1).$ 14. $(mx - n)(px^2 + qx - r).$

15. $(mx - n)(ax^2 - cx - b).$ 16. $(px - q)(3x^2 - cx - b).$

17. $(x^2 - px + q)(ax^2 + bx - c).$ 18. $(2x + 3c)(x^2 + ax - 2b).$

19. $(2x + 3c)(x^2 - 2ax + 3b).$ 20. $(ap - bq)(2p^2 + 3pq + q^2).$

21. $(ap - bq)(3p^2 - pq - 2q^2).$ 22. $(ax + b)(cx^2 + dx + c).$

23. $(ax + b)(2cx^2 - dx - 3c).$ 24. $(3y - ab)(3y - bc)(3y + 5).$

EXERCISE XLVII. (Page 118.)

1. $(6x - y + 1)(x - 6y - 1).$ 2. $(3x + 2y + 1)(2x - 3y - 1).$

3. $(4a + 5b + 4)(3a - 4b - 5).$ 4. $(x - y + 3z)(x + 2y - z).$

5. $(3x + y + 3)(x - 3y + 9).$ 6. $(2a - 5b + 6c)(3a + 4b - 8c).$

7. $(3a - b - 7)(4a - 3b + 8).$ 8. $(7x - y - 1)(x - y + 3).$

9. $(a + 3y)(a - 4y - 5).$ 10. $(2x - 5y - 7z)(2x + 3y + 3z).$

11. $(3x + y - 4z)(3x - 3y - 2z).$ 12. $(3x - 2y + 3z)(2x - 3y + 2z).$

13. $(5x - 3y + 2z)(x - y - z).$ 14. $(a - 2b + 3c)(14a - b - c).$

15. $(2a - b - 3c)(4a - 3b - c).$ 16. $(1 - 3x + 4y)(1 + 7x - 5y).$

EXERCISE XLVIII. (Page 122.)

7. 8.   8. $-8a^3$.   9. -205.   10. 1.

11. $a^3 + pa^2 + qa + r$.   12. -36.   13. 1555.

14. $x + 2$, $x - 3$.   15. $(x + 1)(3x + 2)(2x - 1)$.

16. Last term should be $52a^3$.   18. $(x - 2)(x - 5)(x + 7)$.

19. -535.   20. -800.   21. 101.   22. 115.

23. $-a^3 - pa^2 - qa - r$.   24. $abc - 4ab(a + b)$.

29. -1.   32. 2.   34. $2(a + b)^3$.   35. 0.   39. 0.

40. 0.   41. yes; put 1 for $x + y$.   42. 0.

44. $a^2 + pa + q$, $a^2 + p'a + q'$.

EXERCISE XLIX. (Page 126.)

5. $(p - 1)^2 - (p - 1)(q + 1) + (q + 1)^2$; $a^n - b^n$.

6. $x^{10} - x^5 a^5 + a^{10}$; $1 - (a - b) + (a - b)^2 - (a - b)^3$.

7. $x^2 - 1 + 1 \div x^2$; $x^8 - 2x^6 + 3x^4 - 2x^2 + 1$.

8. $x^2 + y^2 \pm xy$; $(a^2 + 4b^2)(a + 2b)(a - 2b)$; $2x(x^2 + 12y^2)$.

9. $(a + b)(a^2 + b^2 \pm ab)$;
   $x^2 - 2y^2$ and $x^8 + 2x^6 y^2 + 4x^4 y^4 + 8x^2 y^6 + 16y^8$.

10. $(a^2 + bc)(a^4 + 7b^2c^2 - 4a^2bc)$; $(x + 1)^5 (x - 1)^5$;
    $(x + 1)(x - 1)(x^2 + 1)(x^4 + 1)$.

11. $a^3 - (2b^5)^3 = \{a - 2b^5\} \{a^2 + 2ab^5 + 4b^{10}\}$; $(a - b)ab$.

12. Expression $= (4a^2 - 9b^2)(x^3 - 8a^3)$, etc.; $\left(a - \dfrac{1}{2a}\right)\left(a^2 - 1 + \dfrac{1}{4a^2}\right)$.

15. Expression $= (x^2 - y^2)(x - y)^2 = 128a^3b^3(a^2 + b^2)$.

16. $x - 1$, factor dividend.

17. $(a^4 - a^2x^2 + x^4)(a^8 + a^4x^4 + x^8)$.

19. Factor and divide by $a + 1 \div a$, $\therefore a^2 - 1 + 1 \div a^2 = 0$,
    $\therefore a^2 + \dfrac{1}{a^2} = 1$, $\therefore a^2 + \dfrac{1}{a^2} + 2 = 3$, etc.

20. Expression $= (1 - x)(1 - x^n)$.

21. Divisor $= (x - 1)(x^2 - x + 1)$ and given expression vanishes
    for each of these factors.

22. $(x - y)(x^2 + y^2)(x^4 + y^4)$.

EXERCISE L [a]. (Page 129.)

1. $3(x + y)(y + z)(z + x)$. 2. $(a - b)(b - c)(a - c)$.
3. $(a - b)(b - c)(a - c)$. 4. $(a - b)(b - c)(c - a)$.
5. $3(x - y)(y - z)(z - x)$. 6. $3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$.
7. $(x + y)(y + z)(z + x)$.
8. $-(a - b)(b - c)(c - a)(a + b + c)$.
9. $(a - b)(b - c)(c - a)(a + b + c)$.
10. $(x + y)(y + z)(z + x)$. 11. $3(a^3 - b)(b^3 - c)(c^3 - a)$.
12. $-(a - b)(b - c)(c - a)(a^2 + b^2 + c^2 + ab + bc + ca)$.
13. $(a + b + c)(a^2 + b^2 + c^2)$. 14. $(x + y + z)^3$. 15. $(a + b + c)^3$.
16. $6abc$. Insert in text $-(a + b)^3$, and read $-$ before $(c + a)^3$.
17. $(a - b)(b - c)(c - a)(ab + bc + ca)$.

[b.] (Page 130.)

2. By symmetry ; or formula (II) (4), p. 85.
3. By symmetry ; or transpose $3abc$, then a is a factor, etc.
4. $5(x + y)(y + z)(z + x)(x^2 + y^2 + z^2 + xy + yz + zx)$.
5. $(a + b - c)(a^2 + b^2 + c^2 - ab + bc + ca)$;
   $(-a + b + c)(a^2 + b^2 + c^2 + ab - bc + ca)$;
   $(-a - b + c)(a^2 + b^2 + c^2 - ab + bc + ca)$.
6. $12\frac{4}{7}$; 2d term should be $13x^2$.
7. Use synthetic division ; $4m - 12 = 0$, $m = 3$.
8. Given expression $= (x - 3)(x + 2)(x - 5)$, which is true for
   all values of x, $\therefore$ coefficient of *like powers* of x are equal ;
   i. e., $a = -6$, $b = 1$, $c = 30$.
9. $b = -6$, $c = 5$, $a = 12$. 11. $1 \div (a + b + c)$.
12. 2 ; dividend is 2 (divisor). 13. $16abc(b - c)(a - c)(a - b)$.
15. $-abc(b - c)(c - a)(a - b)$.
16. $-abc(a + b + c)(b - c)(c - a)(a - b)$.

EXERCISE LII [a]. (Page 133.)

1. ax ; $x + 2$; $2(x - y)^2$. 2. $2(x - a)$; $ab(x - a)(x - b)$.
3. $x + 3$; $x + 9$. 4. $x - 2$; $x + 2$.
5. $a + b$; $x + 1$; $x - 3$. 6. $x + 5$; $x + 4$; $(x + 1)^2$.
7. $x + 3$; $x - 11y$. 8. $3(x + 3y)$; $(x + y)^2$; $x^2 + 4$.

9. $x + 2$; $x + a$.   10. $x - a$; $x - y$.

11. $x + 3z$; $a + 3$.   12. $a - 1$; $x - a - 4$.

13. $a + b - c$; $a + b + c$.   14. $a + b + x + y$; $x + a$.

15. $x - a$; $8(x - 3y)$; $x + a$. 16. $x + a$; $x - 5$; $(1 - x)^2$.

17. $x^2 + xy + y^2$; $x^2 + a^2$; $x^2 - y^2$.

18. $3(x - y)(x + y)$; $3(a + b)(a^2 + b^2)$.

19. $5(p - q)(p + q)$; $x + y$.

[b.]

20. $x + y$.   21. $3x + 1$; $5x - 1$.

22. $2a + 5$; $a + 5$.   23. $x + 3$; $(x - 1)^2$.

24. $a^2 + ab + b^2$; $a + b$.   25. $2x + 1$; $x^2 + y$.

26. $x^2(3x + 2)$.   27. $x^2 + 2y^2 - 2xy$.

28. $3x + 4a$; unity.   29. 4 ; $1 + \dfrac{1}{a}$.

30. $a^2 + \dfrac{1}{a^2}$.   31. $2x - 1$.

32. $2a + 3b - c$.   33. $5(x + 2y)$.

34. $mx + m - x$.   35. $ap - bq$.

36. $x + 2ab$; omit a in ax^3. 37. $(a - b)(x + a)$.

38. $3(2a - 7)$.

39. $(2ax - y)$; last term of 2nd expression should be $3y^3$.

40. $(x - 1)^2$.

EXERCISE LIII [a]. (PAGE 141.)

1. $2(x + 1)^2$.   2. $x - 5$.   3. $2x^2 - 3x - 1$.

4. $7a^2 + 3a - 1$.   5. $y^2 + 8y - 2$.   6. $y^2 - 3y - 5$.

7. $x^2 - 2x - 3$.   8. $x^2 - 3$.   9. $5x^2 - 1$.

10. $3x^2 - 2xy + y^2$. 11. $x - 1$.   12. $(x + 2)^2$.

13. $x(2x^2 + 2xy - y^2)$.   14. $x - 2$.

15. $(x - 1)(x + 1)$ or $x^2 - 1$.

[b.]

1. $x^2 - 3x + 2$.   2. $x^2 - 13x + 5$.

3. $a^2 - 8$.   4. $(x - 3)$.

5. $12x + 5$.   6. $2x^3 - 4x^2 + x - 1$.

7. $x^2 \dotplus 3x + 5$.

8. $(x + 1)(x^2 + 1)$.

9. $a^3 - a^3 - a - 1$.

10. $2y^2 - 7$.

11. $2x^2 - 3x - 1$.

12. $x^3 + x^2 - 5x + 3$.

13. $3x - 5a$.

14. $2x^2(2x + 9)$.

15. $x^2 - 7x - 3$.

EXERCISE LIV [a]. (Page 143.)

1. $2 \cdot 2a^3$; $2x^3 \cdot 6yz$; $ab \cdot cx^2y$; $a \cdot 12ab^2$; $4z \cdot 7x^2yz$.

2. $42a^3b^3$; $4y^2 \cdot x^4$; $y \cdot 10x^4y^3$; $3c \cdot 7a^2b^3c^2$.

3. $y \cdot x^3y^2$; $2 \cdot 3a^2b^3$; $7a^2 \cdot 3b^2 \cdot 2$; $ab \cdot c$.

4. $2y \cdot 3x^2y \cdot 2$; $8y^3 \cdot 7x^4y$; $2 \cdot 3x^2y^2 \cdot 2xy$; $ax^2 \cdot 6xy$.

5. $2 \cdot 3abc \cdot 2$; $3 \cdot 4x^2y \cdot 5y$; $p \cdot 6pq^2 \cdot p$; $ax^n \cdot by^n$.

6. $1 \cdot a^3b^2c^4$; $xy^3 \cdot 15a^2x^2y$; $n^2 \cdot 3m^3p^4$; p^nq^n.

7. $3x(a - x)$; $3a^2b(a + b)$; $a(a - b^2)$; $abc(a^2 - c^2)$.

8. $4a^2x(a + x)$; $21(x + y)(a + b)$; $a(p + q)(p - q)$.

9. $a(a+b)(b+c)$; $x(x+1)$; $x^2(x-3)$; $(x-1)(x+1)$; $a^3(a-b)^3$.

10. $ab(x + a)(x + b)$; $ab(x^2 - a^2)$; $(x - 1)(x + 1)$.

11. $(x - 2)(x - 1)$; $21(x - 2)(x+2)$; $x(x+1)(x+2)$; $(x+y)^3$.

12. $(x + 1)(x + 2)(x + 3)(x + 4)$; $(a + b)(a - b)$.

13. $x(3x - 2)(2x - 5)(x + 7)$; $x^2y(a^2 - b^2)$.

14. $x^3(x^2 - a^2)$; $x^3(x - a)(x - b)(x - c)$.

15. $6(x - y)^3$; $6(x + y)^5$; $(x + b)(x^2 - a^2)$.

16. $(2x - 5)(9x^2 - 1)$; $a^4(a + 3)^2$; $(a - b)^2(a + b)^3$.

17. $(a + b)^2(a - x)^2$; $(a + b)^2(a^3+b^3)$.

18. $(a - x)^2(b - y)^3$; $6(a + 1)^2(a - 1)^2$.

19. $-(1 - 2x)(1 + 2x)$; $(x^2 - y^2)^2$; $a^nb^n(a - 1)(b + 1)$.

20. $-6a^nb^n(b^2 - 1)$; $x^2 - y^2$; $(x^2 - 1)(x^2 + 1)$ or $x^4 - 1$.

[b.] (Page 144.)

21. $ab(4a^2 - 1)$; $6x(3x - 1)$; $x(x - 2)(x + 2)$.

22. $(x + 1)(x - 1)^2$; $(x - 1)(x - 2)(x - 4)$;
$(x - 1)(x + 2)(x - 3)$.

23. $(x + 3)(x + 4)(x + 5)$; $(x - 1)^2(x + 2)$.

24. $(a - 1)(a - 2)(a + 2)$; $(x^2 - y^2)^2$ or $(x - y)^2(x + y)^2$.

25. $(x + 2) (x - 4) (x - 10) (x + 12)$; $(x + 3) (x - 3) (x - 12)$.

26. $(x - 2) (x - 4) (x - 7)$; $(x + 1) (x + 3) (x - 4)$.

27. $(x + a) (x^2 - b^2)$; $(1 - x) (1 + x)^2$.

28. $(x - a) (x + a) (x - b)$; $(a + b - c) (a + b + c)^2$.

29. $(x - 2)^2 (x + 2)^2$; $(x + 3) (x - 3) (2x - 1)$.

30. $(y + 2) (y - 3) (3y + 1)$; $(2x + 3) (2x - 3) (3x - 2)$.

31. $(2x + 3y) (2x - 3y) (3x - 2y)$; $3 (x - 1) (3x - 1) (3x + 2)$.

32. $20a^2y (4a - 1) (5a + 1) (3a + 1)$; $(4a - 1) (4a + 1) (5a + 1)$.

33. $(x + 2) (x - 2) (3x - 7)$; $x^{3n} - y^{2n}$.

34. $(x - 2) (x - 3) (x - 4) (x - 5)$; $2ay^2 (4x - 1) (x + 3) (3x - 2)$.

35. $(x + y) (x - y) (x^2 + y^2)$; $12 (x^2 - 1)^2$.

36. $(x - a)^2 (x + a)^2$ or $(x^2 - a^2)^2$; $- 12x (x - 1) (x + 1)^3$.

37. $bc (a^2 - b^2)$; $x^2 (x - 1)^2 (x + 1)^2$.

38. $(x + y) (x^2 + xy + y^2) (x - y)$ or $(x + y) (x^3 - y^3)$; $24 (1 - x^4)$.

39. $a^4x^4 - b^4y^4$; $a^5 - b^5$.

40. $12a^3 (a^2 - y^2)$, $(x^3 + a^3)$.

41. $1 - x^3$; $(1 - 2x)^2 (1 + 2x + 4x^2)$; $(x + y)^3 (x^3 + y^3)$.

42. $(x + 1) (x^2 - x + 1) (x^2 + x + 1)$ or $(x + 1) (x^4 + x^2 + 1)$;
    $(x - y) (x^4 + x^2y^2 + y^4)$.

43. $(x^2 - y^2) (x^4 + x^2y^2 + y^4)$ or $x^6 - y^6$; $(x + 4)^3 (x^2 - 4x + 16)$;
    $(x^n - 1) (x^n + 1)$.

44. $1 + x^2 + x^4$; $(x^2 - y^2) (x^4 + x^2y^2 + y^4)$ or $(x^3 - y^3) (x^3 + y^3)$
    or $x^6 - y^6$.

45. $- (a - b) (b - c) (c - a)$; $- (y^2 - z^2) (x^2 - y^2) (x^2 - z^2)$.

EXERCISE LV. (Page 145.)

1. $(x + 1) (x + 2) (x + 3) (x + 5) (x - 5) (x - 6)$.

2. $(x + 1) (x^2 + 1) (6x^3 + 5x^2 + 2x - 1)$.

3. $(x + 2) (x + 3) (x + 4) (x + 5)$.

4. $(x + 3) (x + 4)^2 (x + 5)$.

5. $3 (2x + 3) (2x + 5) (x^2 - x - 4)$.

6. $x (x - 1) (x - 4) (1 - 2x) (1 + 2x)$.

7. $6 (x - 1)^2 (x - 2) (x + 2) (x - 4)$.

8. $(x + 4) (x^2 - 2) (4x^2 + 2x + 5)$.

9. $(x + 5) (x^2 - x + 1) (3x^2 + 5x + 6)$.

10. $a(a + 5)(7a + 9b)(4a^2 + 3a + 9)$.

11. $(x - 1)(2x + 3)(3x - 2)(x^2 - x + 1)$.

12. $(x - 1)(x - 3)(x + 3)(x^2 - 3)(x^2 - 2x + 3)$.

13. $(3x^2 + 2x + 1)(3x^3 - 2x^2 + 2x - 1)(2x^5 + 3x^2 - 2x + 1)$.

14. $(x + 1)(x + 2)(x + 3)(x + 4)$.

15. $(x + 1)(x - 3)(3x^2 - 4x + 6)(3x^2 - 6x + 4)$.

EXERCISE LVI [a]. (Page 146.)

1. $- 30$. 2. 20. 3. $a = b = 12$. 4. 10.

5. $a = - 10$ and $b = - 1$. 6. $- 114$. 7. 13.

8. $c = 22$, $a = 48$. 9. $1 - a$. 10. $b = 2$. 11. $c = b^2(1 - a)$.

13. Divisor $= (x - ay)(x - bz)$, and dividend vanishes for each of these factors; *i.e.*, for $x = ay$, $x = bz$, substitute and subtract.

15. Substitute $x = -a$ in each expression; subtract and $a = p - 1$; substitute this in $a^2 - qa + 1 = 0$.

[b.]

1. Remainders on dividing by $x + c$ is zero. First quotient $= (x + a - c)$, which multipled into $x^2 + a'x + b'$, gives required expression.

6. Unity. 7. $3x^2 + 2x + 1$.

8. $2x(4x^2 + 1)(5x^2 - 1)^2(5x^2 + x + 1)$.

EXERCISE LVII [a]. (Page 153.)

1. $\dfrac{a + b}{a}$; $\dfrac{b - a}{b}$; $\dfrac{x + 1}{x}$; $\dfrac{x - 1}{x}$; $\dfrac{3x + 2}{x}$; $\dfrac{xy + 1}{x}$; $\dfrac{x^2 y - 1}{x}$.

2. $\dfrac{x^2 + x + 1}{x}$; $\dfrac{a^2 - a - 1}{a}$; $\dfrac{a^4 + a^3 + 1}{a^2}$; $\dfrac{6x^3 + 4x^2 - 3}{2x}$.

3. $\dfrac{2x + 1}{x}$; $\dfrac{2x^2 + x - 2}{x}$; $\dfrac{3x^3 - x^2 + 2}{x}$; $\dfrac{3a^2 b^2 - 1}{ab}$.

4. $\dfrac{x}{x - 1}$; $\dfrac{3x - 5}{x - 1}$; $\dfrac{x^2(4x^3 + 3)}{x^3 + 1}$; $\dfrac{x(x^3 + 1)}{x^2 + 1}$.

5. $\dfrac{(a + b + c)(a + b - c)}{2ab}$; $\dfrac{(b + c - a)(a - b + c)}{2ab}$; $\dfrac{x^3 + 1}{x - 1}$.

6. $\dfrac{x(x + 3)(x + 5)}{x + 2}$; $\dfrac{x^2 - a^2 + xy + ay + 1}{x + a}$; $\dfrac{3x^2 + 2x + 1}{x + 4}$.

7. $\dfrac{22a^2 - 40ab + 16b^2}{5a - 6b}$; $\dfrac{a^3 + x^3 + a^2 - x^2}{a + x}$.

8. $\dfrac{2x^4 + x^2y^2 + y^4}{x^2 - xy + y^2}$; $\dfrac{x^2 + xy + y^2}{x + a}$; $\dfrac{x^3}{x - 1}$.

9. $\dfrac{3x^4 - 24x^2 + 3a^2}{a^2 + 2ax + x^4}$; $\dfrac{(a - 2b)(a + 2b)(a^2 + 4b^2)}{(a - 3b)(a + 3b)}$; $\dfrac{8x^3 - 27}{2x - 7}$.

10. $\dfrac{a^3 - b^3}{a + b}$; $\dfrac{x^3 + y^3}{x - y}$; $\dfrac{x^3 - y^3}{x + y}$;

$\dfrac{2x(1 - 3x^2 + 3x - 9x^4 + 9x^5 + 27x^6 - 27x^7)}{1 + 3x}$.

[b.]

1. $a + \dfrac{1}{a}$; $a - \dfrac{1}{a}$; $x + \dfrac{y}{x}$; $1 + \dfrac{b^2}{a^2}$; $a + \dfrac{a^2}{b^2}$; $2 + \dfrac{a}{x + a}$.

2. $1 + \dfrac{3}{x - 2}$; $1 + \dfrac{7}{x - 4}$; $1 + \dfrac{2}{a - 5}$; $1 + \dfrac{y^2}{x^2 - 2y^2}$;

$1 - \dfrac{2}{a^2 + 1}$; $a - 1 - \dfrac{1}{a + 1}$.

3. $x^2 + x + 1 + \dfrac{2}{x - 1}$; $x + a - \dfrac{a^2}{x - a}$; $x - a + \dfrac{a^2}{x - a}$;

$1 + x + \dfrac{x^2}{1 + x}$.

4. $x - \dfrac{4a^2}{x + 2a}$; $1 + \dfrac{2}{x^2 - 3x - 1}$; $1 + \dfrac{a}{2x^2 - 3x + 1}$;

$x^2 - ax + a^2 - \dfrac{5a^3}{x + a}$.

5. $1 - \dfrac{2}{x + 1} + 1 + \dfrac{2}{x - 1}$; $1 + \dfrac{1}{x - 4} + 1 + \dfrac{1}{x - 5}$;

$1 - \dfrac{4x^2 + 3x - 7}{x^3 + 4x^2 - 5}$.

6. $1 + \dfrac{a + b - c - d}{mx + c + d}$; $1 + \dfrac{b - c}{mx - b - d}$; $1 + \dfrac{2}{ax + m - 1}$;

$3x + 1 - \dfrac{11}{6x - 9}$.

7. $1 + \dfrac{2}{ax + m - 2} + 1 + \dfrac{2}{ax + n - 2}$;

$x + 2 + \dfrac{1}{x - 8} + x + 2 + \dfrac{1}{x - 6}$.

8. $12x - 25 + \dfrac{245x - 49}{5x^2 + 9x - 2}$; $x^2 - xy + y^2 - \dfrac{2y^3}{x + y}$;

$a - 6b - c - 1 + \dfrac{16b^2 + 8bc + b}{a + 2b + c}$.

EXERCISE LVIII [a]. (PAGE 156.)

1. $\dfrac{a}{b}$; $\dfrac{c}{d}$; $\dfrac{a^2}{b^2}$; $\dfrac{1}{c}$; $\dfrac{x}{y}$; $\dfrac{ab}{c}$; $\dfrac{x^2}{a^2}$.

2. $\dfrac{ax}{b}$; $\dfrac{x}{4y^2}$; $\dfrac{4b}{5a}$; $\dfrac{2x}{3a^2y}$; $\dfrac{1}{3abc}$.

3. $\dfrac{1}{3bx}$; $\dfrac{x^2}{a^{n-2}}$; $\dfrac{y^n}{3x}$; $\dfrac{a^{m-2}}{b^{m-2}}$; $\dfrac{a-b}{ab}$.

4. $\dfrac{a}{y-x}$; $\dfrac{b}{2a+3c}$; $\dfrac{x+y}{x-y}$; $\dfrac{2a-3b}{3a-2b}$; $\dfrac{a}{a-b}$.

5. $\dfrac{x}{x^2-1}$; $\dfrac{1}{y(a+x)}$; $\dfrac{1}{y}$; $\dfrac{1}{a^2-b^2}$; $\dfrac{a+b}{a-b}$.

6. $\dfrac{a+b}{a^2+ab+b^2}$; $\dfrac{x^2-x+1}{x^2+2x+1}$; $\dfrac{a^2+ab+b^2}{a+b}$; $\dfrac{x}{x-8}$; $\dfrac{x-3}{x-5}$.

7. $\dfrac{x+4}{x+7}$; $\dfrac{x+7}{x-3}$; $\dfrac{x+7a}{x-3a}$; $\dfrac{x+4}{x+3}$.

8. $\dfrac{x^2+x+1}{x^2-2x+1}$; $\dfrac{x+b}{x+c}$; $\dfrac{x+b}{x-c}$.

9. $\dfrac{3(a+7b)}{4(a+5b)}$; $\dfrac{a+b-c}{a+b+c}$; $\dfrac{a^2-b}{b^2-a}$; 1.

10. $\dfrac{2x+a+b}{2x-a-b}$; y in numerator should be b and in denominator x;

$\dfrac{1}{a-b}$; $\dfrac{a+b}{a-2b}$; $\dfrac{5a^2+3b}{a^2-b}$.

11. Irreducible ; $\dfrac{y+z}{z-y}$; $\dfrac{3x+2}{4x+5}$; $\dfrac{(a+b)^2}{a-b}$.

12. $\dfrac{x^2-xy+y^2}{x^2-2xy+y^2}$; $\dfrac{a+b}{a-b}$; $\dfrac{x^2+1}{x^2-1}$.

[b.]

1. $\dfrac{x-11}{4x^2-5x-5}$; irreducible ; $\dfrac{x+4}{(x-1)^2}$.

2. $\dfrac{7x - 2y}{5x^2 - 3xy + 2y^2}$; $\dfrac{x^2 + x - 2}{x^2 + 5x + 5}$; $\dfrac{5a^3(a + x)}{x(a^2 + ax + x^2)}$;

  in first term of numerator, x should be a.

3. $\dfrac{9(x^2 + y^2 + z^2 - xy - xz - yz)}{2x - y - z}$; $\dfrac{ax + by}{ax - by}$.

4. $\dfrac{x + 2}{2x - 1}$; $\dfrac{(x + 1)(3x - 7)}{(x - 1)(7x + 3)}$.

5. $\dfrac{a - b}{1}$; $\dfrac{(2x - 3a)^2}{1}$; $\dfrac{x^3 - 2x^2 + 12x - 18}{x^3 - 2x^2 + x + 4}$.

6. $\dfrac{3(x - 3a)(x - 4a)}{2(x + 3a)(x + 4a)}$; $\dfrac{a(x + 8a)}{x(x + 7a)}$.

7. $\dfrac{3ax^2 + 1}{4a^2x^4 + 2ax^2 - 1}$; $\dfrac{c}{1}$.

8. $\dfrac{a^2 + b^2}{a}$; $\dfrac{x - 5}{x + 5}$; irreducible.

9. $\dfrac{a^2 + b^2 + c^2 - ab - bc - ca}{a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}$; $\dfrac{x + y + z}{2}$.

10. $\dfrac{1}{3(a + b + c)}$; Expre'n $= \dfrac{(x - 1)(x^{n-1} + x^{n-2} + \ldots + x - n)}{(x-1)\{nx^n - (x^{n-1} + x^{n-2} + \ldots + 1)\}}$

  $= \dfrac{(x - 1)(x^{n-1} - 1 + x^{n-2} - 1 + \ldots + x - 1)}{(x - 1)(x^n - x^{n-1} + x^n - x^{n-1} + \ldots x^n - 1)} =$

  $\dfrac{(x - 1)^2 \{x^{n-2} + 2x^{n-3} + 3x^{n-4} + \ldots + (n - 1)\}}{(x - 1)^2 \{x^{n-1} + x^{n-2}(x + 1) + x^{n-3}(x^2 + x + 1) + \ldots + 1\}}$

  $= \dfrac{x^{n-2} + 2x^{n-3} + 3x^{n-4} + \ldots + (n - 1)}{x^{n-1} + x^{n-2}(x + 1) + x^{n-3}(x^2 + x + 1) + \ldots + 1}$

  $= \dfrac{\text{last numerator}}{nx^{n-1} + (n-1)x^{n-2} + (n-2)x^{n-3} + \ldots + 1}$.

11. $\dfrac{(a + b)(b + c)(c + a)}{1}$; $\dfrac{x + 2y + 3z}{x - 3y - 4z}$.

12. $\dfrac{5xy(x + y)}{2(x^2 + xy + y^2)}$; $\dfrac{a^2 + b^2 + c^2 + ab + bc + ca}{5(a^2 + b^2 + c^2 - ab - bc - ca)}$.

EXERCISE LIX [a].　(PAGE 162.)

1. $\dfrac{3}{x}$; $\dfrac{1}{x}$; $\dfrac{a}{x}$; $\dfrac{x + y}{a + b}$; $\dfrac{12}{y}$.

2. $\dfrac{4(a + b)}{5}$; $\dfrac{2(a - b)}{7}$; $-\dfrac{2b}{b} = -2$.

3. $\dfrac{a+b}{ab}$; $\dfrac{b-a}{ab}$; $\dfrac{a^2-x^2}{ax}$; $\dfrac{a+b}{abx}$; $\dfrac{a(a+x)}{x^2}$; $\dfrac{a(a-1)}{x}$.

4. $\dfrac{5x-17}{6}$; $\dfrac{x+17}{6}$; $\dfrac{12a-5}{35}$; $\dfrac{2a^2b-1}{10b^2}$.

5. $\dfrac{7x+2}{12}$; $\dfrac{x(b-a)}{ab}$; $\dfrac{ab+bc+ca}{abc}$; $\dfrac{a^2b^2-c^2}{abc}$.

6. $\dfrac{a+b+c}{abc}$; $\dfrac{x^2+y^2+z^2}{xyz}$; $\dfrac{bcx+acy+abz}{abc}$; $\dfrac{x^2+18x-27}{9x^2}$.

7. $\dfrac{6a+4b+3c}{12x}$; $\dfrac{x(az+ay+yz)}{ayz}$; $\dfrac{a+bx+cx^2}{x^n}$; $\dfrac{x^2-a^2b^2}{abx}$.

8. $\dfrac{2x}{x^2-1}$; $\dfrac{2b}{a^2-b^2}$; $\dfrac{2x^2}{x^2-a^2}$; $\dfrac{a^2+x^2}{a^2-x^2}$.

9. $\dfrac{11}{x^3+3x-28}$; $\dfrac{c-a}{(a+b)(b+c)}$; $\dfrac{x+4}{x^2+5x+6}$; $\dfrac{2a(a-1)}{a^2-b^2}$.

10. $\dfrac{2(x^2+1)}{x^2-1}$; $\dfrac{2(x^2+a^2)}{x^2-a^2}$; $\dfrac{2x^2-32x+127}{x^2-17x+72}$; $\dfrac{24x}{4x^2-9}$.

11. $\dfrac{x^2-2xy+y^2}{4y(x+y)}$; $\dfrac{3(2x+7)}{(x-1)(x-2)(x+4)}$; $\dfrac{1+a}{x+a}$;

$\dfrac{2(1+nx)}{(n^2-1)(1-x^2)}$.

12. $\dfrac{2x^3}{x^4+x^2+1}$; $\dfrac{3ax}{x^3+a^3}$; $\dfrac{3ax}{x^3-a^3}$.

13. $\dfrac{2a}{a^2-x^2}$; $\dfrac{2a}{a(a-x)}$; $\dfrac{1}{a}$.

14. 1; $\dfrac{a}{(1-a)^3}$; $\dfrac{2(x^2-ax+a^2)}{x^2-a^2}$.

15. $\dfrac{2x^2-2xy-y^2}{x^2-y^2}$; $\dfrac{4xy}{(x^2-y^2)^2}$; $\dfrac{a+b}{a^2-ab+b^2}$; $\dfrac{a-b}{a^2+ab+b^2}$;

$x^{2n}+2$.

16. $\dfrac{2x-7}{(x-2)(x-3)(x-4)}$; $\dfrac{2(a+x)}{a^2+ax+x^2}$.

[b.]

1. $\dfrac{1-6x^2}{1-4x^2}$; $\dfrac{x}{4x^2-y^2}$.

2. $\dfrac{4a^2+b^2}{4a^2-9b^2}$; $\dfrac{4}{a-b}$.

3. $\dfrac{18}{x^3-27}$; $\dfrac{2x^2}{x^3-8}$.

4. $\dfrac{16x^2+29x-104}{12(x-5)(x+5)}$; $\dfrac{x+3}{x^4-1}$.

5. $\dfrac{x + 11}{(x - 1)(x + 4)}$; $\quad \dfrac{2x^3 - 5x^2y + 10xy^2 + 5y^3}{(x^2 - y^2)^2}$.

6. $\dfrac{a + b + c}{(a + b - c)(a + c - b)(b + c - a)}$; $\quad \dfrac{6}{(x - 1)(x + 1)(x + 3)}$.

7. $\dfrac{-1}{x + 3y}$; $\quad \dfrac{2916}{x^8 - 6561}$. $\qquad$ 8. $\dfrac{8a^7}{x^8 - a^8}$; $\quad \dfrac{a^2}{(a - b)(a - c)}$.

9. $\dfrac{1}{a + b}$. $\qquad$ 10. $\dfrac{1}{x + 2}$; $\quad \dfrac{4(ab - cd)}{a^2 + c^2 - b^2 - d^2 + 2ac - 2bd}$.

11. 1; $\ 0$. $\qquad$ 12. $\dfrac{a}{xyz}$; $\quad \dfrac{x(1 - 3x + 3x^2)}{(1 - x)^3}$.

13. $\dfrac{1 + 2x + 3x^2}{4(1 - x^4)}$; $\quad \dfrac{x + c}{(x - a)(x - b)}$.

14. $\dfrac{4a^m b^m}{a^{2m} - b^{2m}}$; $\quad \dfrac{y^{n-1}z}{(y - z)^n}$.

15. $\dfrac{a^3b + a^2c + a^2d - c}{a^x}$; $\quad \dfrac{8x^2 + 4x - 3}{(x - 1)(x + 1)(2x + 1)}$.

16. $\dfrac{a(a^2 + 2ax + 3x^2)}{4(a^4 - x^4)}$; $\quad \dfrac{2x}{x + y}$.

17. $\dfrac{2 + x^2 - x^3}{2(x^2 + 1)(x^3 + 1)}$; $\ 2$. $\qquad$ 18. $\dfrac{1}{(x + 1)(x + 2)(x + 3)}$.

[c.]

1. $\dfrac{4a^2 - 2ab}{a^2 - b^2}$, ab for a^2b in num. and denom.; $\quad \dfrac{b^2(a^3 - a^2 + b^3)}{a^6 - b^6}$.

2. $\dfrac{16x^{15}}{1 - x^{16}}$; $\quad \dfrac{q(b - a)}{(x - a)(x - b)}$.

3. $\dfrac{(1 - x)(1 + 2a + a^3)}{(1 - a)(1 + a)(1 + a^2)}$; $\quad \dfrac{p(a - c)(x - b) + q(b - c)(x - a)}{(x - a)(x - b)(x - c)}$.

4. $\dfrac{24(x - 9)}{(x - 1)(x - 3)(x - 5)(x - 7)}$;

$\dfrac{-22x^3 + 44x^2 - 180x - 622}{7(x - 1)(x + 2)(x - 3)(x + 4)}$.

5. 0. $\quad$ 6. 0. $\quad$ 7. 1. $\quad$ 8. $a + b + c$.

9. 0. $\quad$ 10. 0. $\quad$ 11. 0. $\quad$ 12. 0. $\quad$ 13. 0. $\quad$ 14. 0. $\quad$ 15. 1.

16. $(x^3 - y^3)$; sign $+$ should be $-$ between given quantities.

17. $\dfrac{a^2 b \left(a^{2x} + b^2\right)}{\left(a^{2x} - a^x b + b^2\right)\left(a^{4x} - a^{3x} + a^{2x}b^2 - a^x b^3 + b^4\right)}$.

18. $\dfrac{a^{4z}\left(a^{2y} - \dfrac{1}{a^{2y}}\right) + a^{2z}\left(a^{2y} + \dfrac{1}{a^{2y}}\right) - (a^{4z} + 1)}{1 - a^{4z}}$.

EXERCISE LX [a]. (Page 170.)

4. $c^2 \div b^2$; $a^2 \div c^2$; $b \div a$; $\dfrac{x^4}{a^4}$; $b^2 \div a^2$; $b + x$.

5. $(a + b) \div (a - b)$; $4a^x \div 15x^6$; $\tfrac{5}{3}a^2 x$; 1.

6. $\dfrac{a^2 - 9}{a^2 - 16}$; 1; $\dfrac{x - y}{x^2 + y^2}$; $(x + 3)(x + 1)$; $1 \div x$.

7. $\dfrac{1}{(x + 4)^2}$; 1; $-\dfrac{a}{b}$; $\dfrac{1}{z} - \dfrac{1}{x}$; $\dfrac{5b}{3a^{2m}c}$.

8. $xy + yz + zx$; $2(a + b + c)$; $ab + bc + ca$.

9. $2 + 2x^2$; $4ab$; $\dfrac{-1}{a(x - a)}$; $\dfrac{n}{m}$

10. $\dfrac{1}{a^2} + \dfrac{1}{b^2} + \dfrac{1}{c^2}$; $\dfrac{2(a^2 + b^2)}{(a^2 - b^2)^2}$; 1; $\dfrac{1 - d}{1 + b}$.

11. $-2x$; $\dfrac{x^2}{y^2}$; $\dfrac{a^{3x-1}b^{4y-1}}{c^{z+1}}$.

12. $\dfrac{ax}{(x - b)^2}$; $\dfrac{x}{x - 3}$; $x - 1$; $(x - a)^5 \div a^2$.

13. $\left(\dfrac{x + 3}{x - 3}\right)^3$; $\dfrac{a}{b}$; $\dfrac{a(a^2 + ax + x^2)}{(a - x)(a + x)^2}$.

14. $\dfrac{x^2 + xy + y^2}{x + y}$; $\dfrac{1}{x^3 + a^3}$; $\dfrac{a^2 - ab + b^2}{(a + b)(a - b)^2}$; $\dfrac{1 - b^n + b^{2n}}{(1 + b^n)(1 - b^n)^2}$.

15. $\dfrac{(x^2 - 1)(x + 1)}{x^2 + x + 1}$; 1; $\dfrac{(x + b)(x - c)}{(x + a)(x + c)}$.

16. $\dfrac{x - 3b}{x - 2c}$; $\dfrac{(x - c)^2}{(x + a)(x - b)(x + b)}$.

17. $\dfrac{x + b}{x + c}$; $(x + 2)$; $\dfrac{1}{x^2} - 1$.

[b.]

1. $\dfrac{1}{(x-y)^2}$; $\left(\dfrac{5ab^2}{2}\right)^m$.   2. 1; $\dfrac{(x^2-1)(x+1)}{(x-7)(x-5)^2}$.

3. $\dfrac{x+a}{x-a}$; $\dfrac{a}{b}$.   4. $\dfrac{a-b}{a+b}$; $\dfrac{2ab^6}{15cd^6}$.

5. $\dfrac{x^2(x-1)}{x^2-4}$; $(100xy)^m$; $\dfrac{a^2}{b^2}+\dfrac{b^2}{a^2}-1$.

6. $\dfrac{a^2(x+13a)}{b^2c(x-2a)}$, put x for a in first term of numerator and a

   for a^2 in second term; $\dfrac{625a^5}{864b^2}$.

7. $\dfrac{a+b-c}{b-c-a}$; 1.   8. $\dfrac{x^4}{x^2+y^2}$; $\left(\dfrac{5bc^3d^2e}{3a^2f^5}\right)^m$.

9. $\dfrac{(x+a)(x^3+a^3)}{(x^2+a^2)^2(x^2+ax+a^2)}$; $\dfrac{1}{(x-4)(3x-2)}$.

10. $\dfrac{1-y}{x}$; 1.   11. $\dfrac{x^2-6x-8}{x-12}$; 1.

12. $\left(\dfrac{x+y}{x-y}\right)^3$; $\dfrac{c-d}{(a+b)^4}$.   13. $\dfrac{(x-a)^2(x+3a)}{(x-c)^2(x+6a)}$; $\dfrac{(5a^{2m})^z}{7^{mz-z}}$.

14. $abcde$; $\dfrac{y^6}{(y-x)(x^2+y^2)}$.   15. 2; 0.

16. 1; $\dfrac{675cd}{8ab}$.   17. $\dfrac{abc}{(a-c+b)(ab+bc+ca)}$.

18. $\dfrac{ax^3(x^2+a^2)}{x^3+a^3}$; $-\dfrac{p^4q^4}{p^4+p^2q^2+q^4}$.

EXERCISE LXI [a].   (PAGE 176.)

1. $\dfrac{4a-3}{16}$; $\dfrac{8a+1}{28}$; $\dfrac{x-3}{6}$; $\dfrac{a^2-bc}{b^2}$; $\dfrac{a^2}{b^4}$; ab.

2. $\dfrac{2(a-b)}{3}$; $\dfrac{16+x}{16-x}$; $\dfrac{6x-20}{12x+5}$; $\dfrac{2x-5}{x-10}$; $\dfrac{12x-8}{12x+9}$.

3. $-\dfrac{1}{b}$; $\dfrac{a+b}{a-b}$; 1; $\dfrac{b+a}{b-a}$; $\dfrac{x}{2+x}$.

4. $\dfrac{a^2}{b^2}$; $\dfrac{24b+9a}{12a+8b}$; $\dfrac{x^2-1}{x^2+1}$; 1

5. $\dfrac{a - ax - 1}{a - ax + 1}$; $\dfrac{1}{a}$; $\dfrac{2ax}{a^2 + x^2}$; $\dfrac{2x - 35}{x^2}$.

6. $\dfrac{x^3 - 2x^2 - 3}{x^3(3 - x)(3 + x)}$; $\dfrac{1}{x}$; $a + x$.

7. $\dfrac{a^3 + x^3 - a^2}{a^3 + x^3 - x^2}$; $\dfrac{12}{(x - 1)(x - 2)}$; $-\dfrac{a}{x}$.

8. $\dfrac{(a - b)^2}{a^2}$; $(a + 1)^2$; $a^2 - ax + x^2$.

9. $\dfrac{2ab}{a^2 + b^2}$; $\dfrac{4ab(a^2 + b^2)}{a^4 + 6a^2b^2 + b^4}$; $\dfrac{(a^2 + b^2)(a^4 + 14a^2b^2 + b^4)}{2ab(3a^4 + 20a^2b^2 + 3b^4)}$.

10. $\dfrac{b - a + 4}{b - a + 5}$; $\dfrac{6a}{1 - 2x}$.

11. $\dfrac{a^2 + b^2}{b - a}$; $\dfrac{ab + bc + ca}{a^2c + ab^2 + bc^2}$.

[b.]

1. $\dfrac{4ab(a + b)}{(a - b)(a^3 + 2a^2b + ab^2 - a^2 - b^2)}$; $\dfrac{4(ax^2 - 4)}{x^2(3x - 2)(3x + 2)}$;

$\dfrac{1 + x}{1 + x^2}$.

2. $\dfrac{1}{(x^2 - y^2 - z^2)(x^2 + y^2 - z^2)}$; $\dfrac{2}{63}$; $\dfrac{a - 3}{(a - 1)(a + 3)}$.

3. $\dfrac{b^2 - ab}{a^2 + ab}$; x^2y^2.

4. $\dfrac{1}{x^3 + 1}$; $\dfrac{x(x^2 + 2)}{x^4 + 3x^2 + 1}$.   5. 1; $\dfrac{1}{a^2b(b - a)}$.

6. 0; $\dfrac{x + 2y + 6}{8x(y + 6)}$.   7. $-x$; $\dfrac{1 + 2x}{2 + 3x}$.

8. $\dfrac{a^6}{(a + x)(a^2 - x^2)}$; $\dfrac{1}{x^3}$.   9. $\dfrac{4a^2b^2}{(a^2 - b^2)^2}$; 1.

[c.]

1. 1; $a + b$.   2. $16x + 11y$.   3. $5x + 11y$.

4. $\dfrac{x - y}{x + y}$; $\dfrac{3 - 3x}{4 + 4x}$.   5. 1.   6. $\dfrac{2}{x}$.

7. $\dfrac{(a-b)}{(a+b)}$. 8. $\dfrac{a}{2x^2}$. 9. $\dfrac{ab}{a+b}$.

10. 2. 11. $-\frac{1}{2}\frac{1}{2}$ or $-\frac{9}{2}\frac{9}{2}$. 12. c.

16. Take the fractions in pairs, thus :

$$\left(\frac{1}{s-a}+\frac{1}{s-b}\right)+\left(\frac{1}{s-c}-\frac{1}{s}\right)=\frac{c}{(s-a)(s-b)}+\frac{c}{s(s-c)},$$

by substituting for $2s$, etc.

17. a. 20. Multiply given relations out and transpose,

$$a+b+c+d=abc+abd+bcd+acd=abcd\left(\frac{1}{a}+\text{etc.}\right)\text{etc.}$$

EXERCISE LXII. (Page 185.)

21. *1.* $1+x+x^2+x^3+x^4+\ldots$

 2. $1+3x+9x^2+27x^3+81x^4+\ldots$

 3. $1-x+x^2-x^3+x^4-\ldots$

 4. $1-3x+9x^2-27x^3+81x^4-\ldots$

 5. $\dfrac{a^2}{x}+\dfrac{a^2b}{x^2}+\dfrac{a^2b^2}{x^3}+\dfrac{a^2b^3}{x^4}+\dfrac{a^2b^4}{x^5}+\ldots$

 6. $x+\dfrac{x^2}{a}+\dfrac{x^3}{a^2}+\dfrac{x^4}{a^3}+\dfrac{x^5}{a^4}+\ldots$

 7. $a+abx+ab^2x^2+ab^3x^3+ab^4x^4+\ldots$

 8. $1-\dfrac{2x}{a}+\dfrac{3x^2}{a^2}-\dfrac{4x^3}{a^3}+\dfrac{5x^4}{a^4}-\ldots$

 9. $1+x-x^3-x^4+x^6+\ldots$

 10. $1+ax+a^2x^2+a^3x^3+a^4x^4+\ldots$

27. $\dfrac{x^3+y^3}{x^3-y^3}=\dfrac{(x+y)^3-3xy(x+y)}{(x-y)^3+3xy(x-y)}=\text{etc.}$

 Substitute from given conditions.

EXERCISE LXIII [a]. (Page 191.)

1. $x=7$. 2. $x=1\frac{1}{2}$. 3. $x=2$.

4. $x=3$. 5. $x=4$. 6. $x=\frac{21}{11}$, read x for $2x$.

7. $x=\frac{1}{2}$. 8. $x=5$. 9. $x=6$.

10. $x=\dfrac{a(1+b)}{a-2}$. 11. $x=\dfrac{3a-b}{2}$. 12. $x=1$.

13. $x = \dfrac{a^2(b-a)}{b(a+b)}$. 14. $x = \dfrac{3a-6}{4}$. 15. $x = \dfrac{2ab}{a+b}$.

16. $x = \dfrac{d(a+c)}{b}$. 17. $x = b$. 18. $x = 9$.

19. $x = -\frac{3}{4}$. 20. $x = 3$. 21. $x = 60$.

22. $x = 1$. 23. $x = -1\frac{1}{12}$. 24. $x = -\frac{2}{7}$.

25. $x = \dfrac{ab}{a+b-c}$. 26. $x = \dfrac{a^2}{b-a}$. 27. $x = -6\frac{5}{6}$.

28. $x = -\frac{7}{18}$. 29. $x = -\frac{3}{2}$. 30. $x = 3a$.

31. $x = \dfrac{bn+dm+amn+cmn}{b+d+am+cn}$. 32. $x = 14$.

33. $x = \dfrac{acn-abn-abm-bcm}{nb-nc-ma-mc}$. 34. $x = 3a$; right mem. sh. be $2\frac{1}{6}$.

35. $x = 7$. 36. $x = 3$. 37. $x = 3$.

38. $x = \frac{1}{2}$. 39. $x = 11$. 40. $x = -6$.

[b.]

1. $x = -3\frac{1}{2}$. 2. $x = -107$. 3. $x = \dfrac{a+b+c+d}{n-m}$.

4. $x = 3$. 5. $x = \dfrac{b(a-b+c)}{a}$. 6. $x = \dfrac{1}{ab}$.

7. $x = 0$ or $-4\frac{1}{3}$. 8. $x = -\frac{7}{10}$. 9. $x = \dfrac{m}{c}$.

10. $x = 15$. 11. $x = 3$. 12. $x = 1\frac{1}{2}$.

13. $x = -2\frac{1}{2}$. 14. $x = -6$. 15. $x = 7$.

16. $x = \dfrac{m}{2}$. 17. $x = \pm 3$ or ∞.

18. $x - 1 = 0$ and $4x^2 + 5x + 3 = 0$. 19. $x = a + b$.

20. $x = 2$. 21. $x = a$. 22. $x = \dfrac{7bc}{9b+4c}$.

[c.]

1. $x = 13$; second numerator should be 3. 2. $x = -9$.

3. $(x-3)(2x-5) = 0$; $x = 3$.

4. $x = 4\frac{1}{2}$. 5. $x = -1$.

6. $x = \dfrac{ab - bc - ca}{c^2}$.

7. $x = \dfrac{a\,(a - b)\,(a - c)}{(a + b)\,(a + c)}$.

8. $x = 0$.

9. $x = 5$.

10. $x\,(b - a) = 0$; whence $x = 0$, unless $b - a = 0$, in that case x may have any finite value.

11. $x = \pm \sqrt{5}$ or 0.

12. $x = c$.

13. $x\,(2x + 5) = 0$; $\therefore x = 0$ or $-2\frac{1}{2}$.

14. $x = a + b + c$.

15. $x = a^2 + b^2 + c^2$.

16. $x = a^4$. (First numerator on right hand should be $x - 1$.)

17. Take in pairs the fractions with like numerators;

$$x = \frac{np\,(c - a) + mp\,(a - b) + mn\,(b - c)}{m\,(a - c) + n\,(b - a) + p\,(c - b)}.$$

18. $(-7x + 49)\left\{\dfrac{1}{x^2 + x - 2} - \dfrac{1}{x^2 + x - 12}\right\} = 0$;

$\therefore x = 7$ or ∞.

19. Complete the divisions, cancel and transpose;

$$\therefore \frac{2}{x - 4} - \frac{1}{x - 5} - \frac{1}{x - 2} = 0;$$

$$\text{or} \quad \frac{1}{x - 4} - \frac{1}{x - 2} = \frac{1}{x - 5} - \frac{1}{x - 4};$$

whence $(x - 8)\,(x - 4) = 0$; $\therefore x = 8$.

The value 4 is not admissible.

20. $x = \dfrac{bn\,(q - p)\,(m - p) + ap\,(q - n)\,(m - n)}{b\,(q - p)\,(m - p) + a\,(q - n)\,(m - n)}$.

21. $x = \{m\,(b - c) - n\,(a + c)\} \div (m - n)$

22. $(a^2x + b^2x - a^2b - ab^2 - b^2c + a^2c) \times$

$$\left\{\frac{1}{(x - a)\,(x - b)} - \frac{1}{(x - a - c)\,(x - b + c)}\right\} = 0.$$

$\therefore x = \{a^2\,(b - c) + b^2\,(c + a)\} \div (a^2 + b^2)$.

23. $(a - b)\left(\dfrac{x}{n - o} - \dfrac{1}{p - q}\right) = 0$, $\quad x = \dfrac{n - o}{p - q}$.

24. $\dfrac{x - 2a}{b + c - a} - 1 + \text{anal.} + \text{anal.} = 0$, whence

$$(x - a - b - c)\left\{\frac{1}{b + c - a} + \text{anal.} + \text{anal.}\right\} = 0;$$

$x = a + b + c$.

25. $\dfrac{a-x}{a^2-bc} - \dfrac{1}{a+b+c} + \&c. + \&c. = 0$, or

$\dfrac{ab+bc+ca-(a+b+c)\,x}{(a^2-bc)\,(a+b+c)} + \text{anal.} + \text{anal.} = 0$;

$x = (ab+bc+ca) \div (a+b+c)$.

EXERCISE LXIV [a]. (PAGE 198.)
PROBLEMS.

1. $10\frac{5}{12}$ dozen. 2. \$36000. 3. 12 years.

4. \$300. 5. $\dfrac{ma-12b}{12-m}$. 6. $\dfrac{abc}{b+c}$.

7. 754. 8. \$3.75. 9. 142857.

10. \$8000. 11. $190\frac{50}{121}$ bushels. 12. 90 and 91.

13. Equation reduces to $(4-4)\,x+40=0$; $x=\infty$; *i. e.*, conditions of problems are inconsistent. In fact, *area* will *always be* 45 ft. less, under the given conditions ; for using 45 for 85, the resulting equation is an identity.

14. $\$1857.35\frac{345}{673}$ and $\$142.64\frac{328}{673}$.

15. 857142 ; x representing number, equation is
    $\frac{1}{10}(x-2)+200,000=\frac{1}{3}x$. 16. \$7.60.

17. 550 ; read 6 in first line ; 4 times and 6 cts. in second line.

18. $13\frac{3}{4}$ feet and $16\frac{1}{2}$ feet.

19. A, \$2800 ; B, \$3900 ; C, \$5138 ; D, \$2196 ; E, \$2966.

20. \$14.

[b.]

1. 960 gallons. 2. 420 acres. 3. \$1280, $7\frac{1}{2}\%$.

4. Gain or loss $\% = \dfrac{q\,(100+n)\sim100p}{p}$, according as

    $q \,\substack{> \\ <}\, \dfrac{100p}{100+n}$.

5. B makes 1740 yds. in 4 m. 34 sec.; C makes 1700 yds. in 4 m. 32 sec. Let $x=$ time in min. from starting at which A overtakes B, then $\frac{x}{4\frac{1}{2}}\cdot1760=20+\frac{x}{4\frac{17}{30}}\cdot1740$, $x=1\frac{641}{622}$ min., distance $775\frac{95}{311}$ yds. from start. Similarly A is found to pass C in $3\frac{2}{5\frac{1}{6}}$ m.; distance $1456\frac{16}{29}$ yds. from start.

6. $10\frac{95}{121}$ miles.   7. 5 gal.   8. \$7400.   9. 57 miles.

10. $mnpqr \div (mnpq - mpq - npq - mnq - mnp)$.

11. 484.   12. 1, 2, and 3.

13. 12000 sq. yds.; 45 cts.   14. 189.

15. $x =$ distance; then $\dfrac{2x}{b} = \dfrac{x}{ap} + \dfrac{px - x}{bp}$.

16. Let $2x$ and x be digits; $(2001x)^2 - (1002x)^2 = 2999997x^2$
$= 4x^2 \times 749{,}999\frac{1}{4}$.   17. 180,000.

18. $\left(\dfrac{n+1}{n}\right)^{n+1}$.   19. $\dfrac{p\,(11m - 21n)}{20\,(m - n)}$.

20. Regular rate 40 miles, diminished rate $38\frac{2}{21}$ miles; 100 miles.

21. $221 : 273 :: 187 : 231$.   22. $\dfrac{ap - an}{m - n}$.   23. $\frac{1}{2}$.   24. 14172.

EXERCISE LXVII. (Page 212.)

1. $x = 3$; $y = 2$.   2. $x = 6$; $y = -4$.

3. $x = 1$; $y = 3$.   4. $x = 5$; $y = 4$.

5. $x = 4$; $y = -3$.   6. $x = -2$; $y = 3$.

7. $x = 1$; $y = 2$.   8. $x = -20\frac{5}{6}$; $y = -19\frac{1}{3}$.

9. $x = 10$; $y = 9$.   10. $x = \frac{511}{303}$; $y = \frac{1775}{1212}$.

EXERCISE LXVIII. (Page 213.)

1. $x = 2$; $y = 3$.   2. $x = \frac{76}{45}$; $y = -\frac{14}{9}$.

3. $x = 3$; $y = -2$.   4. $x = 5$; $y = -5$.

5. $x = 4$; $y = 4$.   6. $x = \frac{232}{71}$; $y = -\frac{79}{71}$.

7. $x = 3$; $y = 2$.   8. $x = 2$; $y = 1$.

9. $x = 4$; $y = -3$.   10. $x = 3\frac{1}{3}$; $y = \frac{1}{2}$.

EXERCISE LXIX. (Page 214.)

1. $x = 5$; $y = 7$.   2. $x = 1$; $y = -1$.

3. $x = 2$; $y = -3$.   4. $x = -2\frac{21}{34}$; $y = 5\frac{13}{34}$.

5. $x = 5$; $y = -4$.   6. $x = \frac{2}{9}$; $y = \frac{1}{6}$.

7. $x = 1\frac{61}{106}$; $y = 2\frac{38}{159}$.   8. $x = 5$; $y = 6$.

9. $x = 1\frac{398}{559}$; $y = -1\frac{59}{129}$.   10. $x = 3\frac{5}{29}$; $y = -3\frac{16}{29}$.

EXERCISE LXX. (PAGE 216.)

1. $x = \dfrac{a+b}{2}$; $y = \dfrac{a-b}{2}$.　　2. $x = \dfrac{1-b^2}{a-b}$; $y = \dfrac{ab-1}{a-b}$.

3. $x = \dfrac{mp-nq}{m^2-n^2}$; $y = \dfrac{np-mq}{m^2-n^2}$.

4. $x = 1$; $y = 1$.　　　　5. $x = a+b$; $y = -1$.

6. $x = \dfrac{b(4b^2-7a^2)}{4b^2-3a^2}$; $y = \dfrac{a(9a^2-4b^2)}{3a^2-4b^2}$.

7. $x = y = \dfrac{2}{a+b}$.　　　　8. $x = m+n$; $y = m-n$.

9. $x = y = \dfrac{m}{a+c}$.

10. $x = \dfrac{a(bc-2ac-c^2-a^2+3ab)}{2(2ab+bc-ac-c^2)}$; $y = \dfrac{a(3a^2+ab+4ac+ac^2-bc)}{2(2ab+bc-ac-c^2)}$.

EXERCISE LXXI [a]. (PAGE 218.)

1. $x = 6$; $y = 12$.　　　　2. $x = 4$; $y = 3$.

3. $x = 7$; $y = 10$.　　　　4. $x = 13\frac{2}{19}$; $y = -4\frac{17}{19}$.

5. $x = 6$; $y = 12$.　　　　6. $x = 4\frac{5}{32}$; $y = -12$.

7. $x = 8$; $y = -\frac{1}{2}$.　　　　8. $x = 2$; $y = 7$.

9. $x = y = 5$.　　　　10. $x = 4$; $y = 5$.

11. $x = 8\frac{1}{2}$; $y = -\frac{1}{2}$.　　　　12. $x = y = \dfrac{a^2b^2}{a^2+b^2}$.

13. $x = \dfrac{2mn(n^2-m^2)}{n^4+6m^2n^2+m^4}$; $y = \dfrac{n^4-m^4}{n^4+6m^2n^2+m^4}$.

14. $x = \dfrac{a^2}{a-b}$; $y = \dfrac{b^2}{b-a}$.

15. $x = 4$; $y = 1$.　　　　16. $x = 6$; $y = -2$.

17. $x = -2$; $y = 4$.　　　　18. $x = 3$; $y = -4$.

[b.]

1. $x = y = \frac{1}{16}$.　　　　2. $x = 14$; $y = -14$.

3. $x = 4\frac{1}{31}$; $y = 11\frac{7}{31}$.　　　　4. $x = 21$; $y = 20$.

5. $x = 3$; $y = 7$.

6. $x = \dfrac{ab - 1}{(1 - a)(1 - b)}$; $y = \dfrac{a - b}{(1 - a)(1 - b)}$.

7. $x = \dfrac{ab + 4b - 2c}{ab - b + 2a}$; $y = \dfrac{a^2 + 2a + ac - ab + b - c}{ab - b + 2a}$.

8. $x = \pm 9$; $y = \pm 3$.   9. $x = a$; $y = b$.

10. $x = 9$; $y = 2$.   11. $x = \dfrac{a}{a - b}$; $y = \dfrac{b}{a + b}$.

12. $x = 8$; $y = 2$.

13. $x = \dfrac{b + c - a - d}{4(bc - ad)}$; $y = \dfrac{c + d - a - b}{2(bc - ad)}$.

14. $x = -2\frac{3}{5}$; $y = -6\frac{1}{5}$.   15. $x = y = 3\frac{1}{2}$.

16. $x = 6\frac{2}{3}$; $y = 8$.

17. $x = \dfrac{b'c - bc'}{ab' - a'b}$; $y = \dfrac{ac' - a'c}{ab' - a'b}$.

1. $x = y = \infty$, and the equations are *inconsistent*; thus, put $\dfrac{a}{a'} = \dfrac{b}{b'} = k$, and $\therefore a = ka'$, $b = kb'$, and substituting these values of a and b in $ax + by = c$, we get $a'x + b'y = \dfrac{c}{k}$, which is inconsistent with the second given equation.

2. $x = y = \frac{0}{0}$, *i. e.* the equations are *not independent*; thus, put $\dfrac{a}{a'} = \dfrac{b}{b'} = \dfrac{c}{c'} = m$. Then $a = ma'$, $b = mb'$, $c = mc'$, and substituting in (1), we get $ma'x + mb'y = mc'$, which is a *multiple* of the second given equation.

EXERCISE LXXII.   (Page 222.)

1. $x = 3$; $y = 6$.   2. $x = \frac{1}{2}$; $y = 1$.

3. $x = \frac{1}{2}$; $y = 1$.   4. $x = -2$; $y = \frac{2}{3}$.

5. $x = -\frac{2}{5}$; $y = \frac{1}{3}$.   6. $x = -\frac{1}{2}$; $y = 1$.

7. $x = y = a + b$.   8. $x = \dfrac{b(a^2 - bc)}{a - b}$; $y = \dfrac{c(a^2 - bc)}{a - c}$.

9. $x = -\frac{1}{18}$; $y = \frac{1}{132}$.   10. $x = y = \dfrac{a^2 + b^2}{ab}$.

EXERCISE LXXIII [a]. (PAGE 223.)

1. $x = y = z = 4$.   2. $x = 1\frac{5}{8}$; $y = 7\frac{1}{8}$; $z = -2\frac{3}{8}$.
3. $x = 14\frac{1}{2}$; $y = -1\frac{1}{2}$; $z = -3\frac{1}{2}$.   4. $x = 2$; $y = -3$; $z = -4$.
5. $x = 18$; $y = -122$; $z = -79$.   6. $x = 4$; $y = 3$; $z = 7$.
7. $x = 4$; $y = 5$; $z = 6$.   8. $x = -1$; $y = -2$; $z = 5$.
9. $x = 1$; $y = 2$; $z = 3$.   10. $z = -1\frac{31}{41}$.
11. $x = 3$; $y = 21$; $z = -3$. 12. $x = 3$.
13. $x = 6$; $y = 8$; $z = 10$.   14. $x = 6$; $y = -2$; $z = -3$.
15. $x = 3$; $y = 6$; $z = 8$.   16. $x = -5$; $y = 9$; $z = -8$.
17. $x = -4\frac{1}{9}$, $y = -4\frac{2}{3}$. $z = -3\frac{1}{108}$.

[b.]

1. $x = \frac{1}{2}$; $y = \frac{1}{6}$; $z = -\frac{1}{3}$. Divide through by xyz in each equation.

2. $x = 5$; $y = 1\frac{1}{4}$; $z = \frac{5}{13}$.

3. $x = 1\frac{7}{24}$; $y = -\frac{2}{3}$; $z = 2\frac{2}{3}$.

4. $x = \frac{1}{a}$; $y = \frac{1}{b}$; $z = \frac{1}{c}$.

5. $x = \dfrac{ab + ca - bc}{2a}$; y and z symmetrically.

6. $z = 3abc \div (c - a)(b + c)$, x and y symmetrically.

7. $x = a - b$; $y = b - c$; $z = c - a$.

8. $x = 2a \div (b + c - a)$; y and z symmetrically.

9. $x = 1 \div (a - b)(a - c)$; y and z symmetrically.

10. $z = \frac{580}{619}$.   11. $x = \frac{1}{2}$; $y = \frac{1}{3}$; $z = \frac{1}{4}$.

12. $x = a^2 - b^2$; $y = b^2 - c^2$; $z = c^2 - a^2$.

13. $z = \frac{1}{2}(a + b + c)(2a + b + c)$; x and y by symmetry.

14. $z = \dfrac{mbpnd + mapnf}{mbp + mna - npc}$; $x = \dfrac{pmn(acb + bdc)}{mnpd + npfc - pmfb}$.

15. $(x + y + z)(a + b + c) = (a + b + c)^2$; $x + y + z = a + b + c$;
$$x = \frac{a^3 + b^3 + c^3 - (a + b)(b + c)(c + a) + 5abc}{a^2 + b^2 + c^2 - ab - bc - ca}.$$

EXERCISE LXXIV [*a*].   (PAGE 226.)

1. $x = 4$; $y = 9$; $z = 16$; $u = 25$.
2. $x = 3$; $y = 0$; $z = 5$; $u = 2$.
3. $x = 5$; $y = 3$; $z = 1$; $u = 4$.
4. $x = -5\frac{2}{11}$; $y = 16\frac{5}{11}$; $z = -5\frac{3}{11}$; $u = 11\frac{7}{11}$.
5. $x = 3$; $y = 2$; $z = -4$; $u = 5$.
6. $x = 0$; $y = -1$; $z = 2$; $v = -4$.
7. $x = -1$; $y = -2$; $z = -3$; $u = -4$; $t = -5$.
8. $x = 1$; $y = 2$; $z = 3$; $u = 4$.

[*b*.]

1. $x = a + b + c$; $y = a + b - c$; $z = a - b + c$; $t = b + c - a$.
2. $x = 6$; $y = -1$; $z = 3$; $u = 2$.
3. $x = \frac{1}{3}(a - 2b + c + d)$; $y, z,$ and u by symmetry.
4. $x = \frac{1}{2}(a - b + c - d + e)$; $y, z, u,$ and v by symmetry.
5. $x = \frac{1}{2}(2a - b - c - e + 2d)$; $y, z,$ and u by symmetry.
6. $x = \frac{1}{2}(a + d)$; $y, z, u,$ and v by symmetry.
7. $x = 30$; $y = 20$; $z = 42$; $u = 72$;   (y should by z in second equation.)
8. $x = a + b + c$; $y, z,$ &c., by symmetry.
9. $x = \frac{1}{5}(a + b + c + d + e - 4f)$; $y, z,$ &c., by symmetry.
10. Divide each side of every equation by xyz; $x = 1 \div b - c$; y and z by symmetry.

EXERCISE LXXV [*a*].   (PAGE 229.)

1. $x = 714285$; $y = 142857$.   2. $x = 40$; $y = 65$.
3. Willie 4;   Charlie 8.   4. $x = 1.234$; $y = 5.678$.
5. $x = 147$; $y = 63$.   6. 76.
7. 13 : 17.   8. 73.
9. 480 gallons;   400 gallons;   560 gallons.
10. \$10260;   \$7560.   11. $\frac{4}{6}$.
12. $10x + y = 6(x + y)$; $\therefore 4x = 5y$; $\therefore 10y + x = 9x = 5x + 5y$, etc.
13. 98 or 89.   14. A, 200 lbs.;   B, 250 lbs.;   C, 350 lbs.

15. 82 apples; gave away 2. 16. $5000; $3000; $4000.

17. 40; 88; 104. 18. 486.

19. $\dfrac{a(b-c)}{b-a}$; $\dfrac{b(c-a)}{b-a}$. 20. $x = \dfrac{2b}{1-a}$; $y = \dfrac{2b}{1+a}$.

21. $\frac{3}{4}$, $\frac{1}{4}$.

22. A, 105; B, $52\frac{1}{2}$; C, 210 minutes; A, B, and C in 30 minutes.

[b.]

1. First, 220 gallons; Second, 100 gallons.

2. 3674. 3. A, $40; B, $24; C, $16.

4. $d(m+n) \div 2mn$; $d(n-m) \div 2mn$.

5. $x+y : x-y : xy :: 5 : 1 : 18$; $x=9$; $y=6$.

6. $x = \dfrac{l(m-1)(q-mr) - (1-mn)(mp-lq)}{m^2 l(1-n) + mnl(1-m) - m(1-mn)}$.

7. 130. 8. 315 miles. 9. 9; $8\frac{2}{11}$ miles per hour.

10. $x = (p+1)n$; $y = (pq-1)n$; $z = (q+1)n$.

11. $x = 1\frac{5}{7}$; $y = 2\frac{2}{5}$; $z = -12$.

12. $\dfrac{c(a+b)}{2ab}$; $\dfrac{c(b-a)}{2ab}$. 13. $\dfrac{qrm}{qr-ps}$ hours.

14. A, $2.60; B, 1.26\frac{2}{3}$; C, $61\frac{1}{3}$ cts. 15. *Ans.* $3\frac{1}{4}$ miles.

16. 3000 ft. from first station; $x =$ distance from first station;

$y = $ A's rate per second; $z = $ B's rate;

then $\dfrac{4000}{y} - \dfrac{4000}{z} = 40$; $\dfrac{x}{y} - \dfrac{x}{z} = 30$;

substitute in this the value of $\dfrac{1}{y} - \dfrac{1}{z}$, viz. $\frac{1}{100}$, from first

equation and $x = 3000$.

17. Gold coin, $2; silver, $1.

18. $11\frac{1}{8}$ miles; 7 miles; and $5\frac{1}{8}$ miles. 19. $5200; $2480.

20. $x = $ time for A, y for B, z for C;

then $\dfrac{1}{y} + \dfrac{1}{z} = \dfrac{m}{x}$, or $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{m}{x} + \dfrac{1}{x} = \dfrac{m+1}{x}$;

$\therefore m+1 = \dfrac{xy + yz + zx}{yz}$, similarly for $n+1$ and $p+1$.

21. x = rate of locomotive, y = rate of coach, z = distance;
   then $6x + 8y = z = 5\frac{1}{2}(x + 1\frac{3}{4}) + 7\frac{1}{2}(y + 1\frac{3}{4})$
   $= 7\frac{1}{12}(x - 1\frac{3}{4}) + 5\frac{1}{12}(y - 1\frac{3}{4}); \quad x = 38\frac{1}{2}, y = 7, z = 287.$
   After should be *before* in first line of equation.

22. Let m, n be the required *dividends*:
   then given fraction $= \dfrac{m}{3 + 4z} + \dfrac{n}{6 + 7z}$.
   Multiply out and equate coefficients, and we get
   $6m + 3n = 27, \quad 7m + 4n = 34; \quad m = 2, \quad n = 3.$

23. $\dfrac{bc - ad}{cf - de}; \quad \dfrac{ab - be}{cf - de}.$ 24. $\dfrac{9}{8x - 7}; \quad \dfrac{6}{5x - 4}; \quad \dfrac{3}{2x - 1}.$

25. 16, 20, 42.

EXERCISE LXXVI.   (Page 239.)

6. $-\frac{2}{3}, -10; \quad \pm(a - b); \quad \pm(1 - a); \quad \pm a + \frac{3}{4}.$

7. $\pm(3a - 2b); \quad \pm\dfrac{a + b}{a - b}; \quad 3, \frac{5}{3}.$

8. $2b, -2a; \quad \pm 123; \quad \pm 275.$ 10. $\pm 2; \quad \pm 2\frac{1}{2}; \quad \pm 2\frac{1}{3}.$

12. $\pm 5; \quad \pm\frac{5}{3}a; \quad \pm a.$ 13. $\pm\sqrt{(mn)}; \quad \frac{89}{63}, \frac{1}{7}; \quad \infty.$

15. $12, 8; \quad \pm\sqrt{\left(\dfrac{a + c}{c - 1}\right)}.$

16. $\pm\sqrt{(ac)} \div \sqrt{(bd)}; \quad \pm\sqrt{(ab)}; \quad \pm 1.$

17. $\pm 1; \quad 2a, 2b.$ 18. $\pm\sqrt{(m^2 + n^2)}; \quad \pm\sqrt{(a^2 + b^2)}.$

19. $\pm\sqrt{(a^2 + 2ab - 3b^2)}; \quad \pm\sqrt{(a^2 + ab + b^2)}.$ 20. $\pm 1.$

EXERCISE LXXVII.   (Page 241.)

1. $11\frac{1}{2}.$ 2. $\pm 385.$ 3. $\pm 13.$

4. $\pm 77, \pm 91.$ 5. $\pm\sqrt{10}.$ 6. 78, 52, 39.

7. 481, 259.   8. $\pm\sqrt{3}, 0.$ 9. 2, 4, 8.

10. $\pm\sqrt{6}.$ 11. 15 or 21.   12. $a \pm \sqrt{(a^2 - b)}.$

13. 40 ft.   14. 8, 12, and $-40, 60.$ 15. 12 ft.

16. 140, 120.   17. 20 per cent.   18. 960, 280.

19. 285, 152.   20. 4 hrs.

EXERCISE LXXIX. (PAGE 250.)

1. $-7, 1$; $6, 2$; $-7, -3$.
2. $-2, -1$; $-m \pm \sqrt{(m^2 + n)}$; $\frac{1}{2}\{m \pm \sqrt{(m^2 + 4n)}\}$.
3. $12, -2$; $-6, -20$; $30, -4$.
4. $23, -1$; $\frac{3}{2}, -\frac{1}{3}$; $\frac{1}{5}, -4$.   5. $2\frac{2}{3}, 1$; $-\frac{1}{4} \pm \frac{1}{4}\sqrt{-543}$.
6. $\frac{15}{13}, 9$; $\frac{51 \pm \sqrt{1721}}{40}$; $-5 \pm \sqrt{2}$.   7. $6\frac{1}{2}, -4\frac{67}{108}$; $3\frac{1}{4}, -2\frac{1}{4}$.
8. $4, \frac{4}{3}$; $2, \frac{39}{5}$; $28, -3$.   9. $\frac{32}{5}$; $-1\frac{1}{6}, 4$.
10. $-\frac{8}{15}, 2$; $-23, 1$.   11. $\frac{1}{2}(-5 \pm \sqrt{65})$; $-2a, a$.
12. $a(a-b), b(a+b)$; $-1, m \div (m-n)$.
13. $56\frac{7}{8}, 12\frac{3}{4}$; $(m+n)^2, -(m-n)^2$.   14. $a+b, a$; $a+b, 0$.
15. $a \pm b$; $-2a, 2b$; $3, a^2 \div (6-a)$.

EXERCISE LXXX [a]. (PAGE 257.)

1. $4, 3$; $-\frac{2}{3}, 1$; $-\frac{7}{4}, 4$; $-2\frac{1}{3}, 2\frac{1}{3}$.
2. $-\frac{7}{4}, 4$; $-5, -\frac{8}{5}$; $3\frac{1}{2}, 5$.   3. $-\frac{3}{2}, 6$; $-\frac{5}{2}, 7$; $\frac{1}{3}, -2$.
4. $30, -4$; $6 \pm \sqrt{-64}$; $\frac{29}{6}, -\frac{5}{3}$.   5. a, b; b, a.
6. $c \div (a-b), d \div (a-b)$; $3, -5$.
7. $2, -3\frac{1}{2}$; $a+b, \dfrac{ab}{a+b}$.

[b.] (PAGE 258.)

NOTE.—*Imaginary roots generally omitted.*

1. $\pm \sqrt{2}$; $\pm 1, \pm 2$; $\pm 2, \pm 3$.
2. $\pm \sqrt{13}$; $\pm 2, \pm \sqrt{10}$; $-\frac{3}{7}, -\frac{6}{7}$.
3. $2, -\sqrt[3]{11}$; $2, 2\frac{1}{4}$.   4. $3, 2$; ± 11.   5. $-2, 4$; 2; 1.
6. $\pm \sqrt{\{+\frac{1}{2}(-3 \pm \sqrt{201})\}}$; $x^2 + 5 = 2$, or -1.
7. $(\pm \sqrt{2}) \div 4$; $-2a, a\sqrt[3]{2}$; $x^2 = \frac{1}{2}\{a^2 \pm \sqrt{(a^4 - 4b^4)}\}$.
8. $-1, 2$; $\pm \frac{3}{4}, \pm \frac{1}{4}\sqrt{11}$; $1, -2$.
9. $\frac{1}{2}\{-7 \pm \sqrt{41}\}, \frac{1}{2}\{-7 \pm \sqrt{21}\}$; $-4, 3, 3 \pm 21$.
10. $\frac{1}{2}\{1 \pm \sqrt{(4a-3)}\}$; $1, 2, 4$.   11. -1; $-1, -2, 2\frac{1}{2}$; 1.
12. $x^3 - 8 + x^2 - 4 = 0$, $x = 2$; 3; $0, -1, -1$.
13. $1, \frac{1}{2}\{p-1 \pm \sqrt{(p^2 - 2p - 3)}\}$; $1, \frac{1}{2}\{2p-1 \pm \sqrt{(4p^2 - 4p - 3)}\}$; $1, x^2 + x + 1 = 0$.

14. $-1, \pm 2$; $a, -a, -b$; $0, \frac{1}{3}, -\frac{1}{2}$.

15. $1, 1, \frac{1}{2}(-3 \pm \sqrt{5})$; $2, \frac{1}{2}, -2 \pm \sqrt{3}$;

     equation is $\left(x + \dfrac{1}{x}\right)^2 + 1\frac{1}{2}\left(x + \dfrac{1}{x}\right) - 10 = 0$, or

     $2y^2 + 3y - 20 = 0 \left(\text{if } y = x + \dfrac{1}{x}\right)$, $(2y - 5)(y + 4) = 0$, etc.

[c.]

1. $7, -2$.   2. $\frac{3}{4}, 3$.   3. $2, -22$.

4. $a + 2c, -\dfrac{a(a + b)}{a + 2c}$.   5. $\pm \sqrt{6}, \pm \sqrt{11}$.   6. $0, -\frac{23}{13}$.

7. $3, -1$.   8. $5, -4\frac{2}{7}$.   9. $0, \pm 6$.   10. $-2\frac{1}{2}$.

11. $\dfrac{\sqrt{\{(a - b)(c + d)\}} \pm \sqrt{\{(a + b)(c - d)\}}}{\sqrt{\{(a - b)(c + d)\}} \mp \sqrt{\{(a + b)(c - d)\}}}$.   Use Art. 133.

12. $-\frac{3}{4}$.   13. $\pm 8, \pm 1$ (put y for $x^2 - 8$ in the denominators).

14. Complete the divisions, transpose and divide by 3, and

     $\dfrac{x^2 - \frac{1}{2} - 9x - 3}{x + 3} = x - 9\frac{1}{2}$, $2\frac{1}{2}x = 25$, $x = 10$.   15. $4, 0$.

16. $(4a - 5b) \div 6ab$, $(a - 2b) \div 3ab$.

17. $\pm \frac{33}{65}\sqrt{-1}, \pm \frac{56}{65}\sqrt{-1}$.

18. Use first quantity for the *unknown*; $7, 1, 4, 4$.   19. $1, 3$.

20. $\pm 3\sqrt{\pm 1}, \pm 2\sqrt{\pm 1}$.   21. $\frac{9}{7}$.   22. $c, c - \frac{1}{2}(a + b)$.

23. Complete the divisions, right member cancels, and

     $\dfrac{1}{x - 9} + \dfrac{1}{x - 25} - \dfrac{2}{x - 49} = 0$, or

     $\dfrac{1}{x - 9} - \dfrac{1}{x - 49} = \dfrac{1}{x - 49} - \dfrac{1}{x - 25}$; $x = 19$.

24. Separate the factor $\dfrac{1}{x + a + b}$; $x = \dfrac{a^2 + b^2}{a + b}$.

25. -1, and $n(x^2 - x + 1) + 1 = 0$.

26. $\frac{1}{2}\{-1 \pm \sqrt{45}\}$ ($+ x$ should be $- x$ in given equation);

     put equation in the form $(x^2 + x)^2 - (x^2 + x) = 132$, etc.

27. $\pm 2a, \pm 2a\sqrt{-1}$.   28. $2, 2\frac{1}{2}$.   29. $4, -\frac{5}{3}$.   30. $8, -\frac{8}{7}$.

[d.]   (Page 260.)

1. $3, \frac{1}{3}, \pm \sqrt{-1}$ (see second equation in [b] 15).

2. Equation is $(x + 1)(x^2 - x + 1 \pm ax)$;

     $x = -1, \frac{1}{2}\{1 - a \pm \sqrt{(a^2 - 2a - 3)}\}$.

3. $-1, -\frac{1}{2}, -2.$ 4. $2, -1, -4.$ 5. $-6, 4.$

6. $-2\frac{1}{2}, \infty.$ 7. $3, \infty.$ 8. $3, \frac{1}{3}, \frac{1}{2}(1 \pm \sqrt{-3}).$

9. Take together pairs of like numerators and equation reduces to $x^4 - 5x^2 + 6 = 0 = (x^2 - 3)(x^2 - 2).$

10. $-4, -2, 3 \pm \sqrt{7}$; observe that one fraction is reciprocal of the other; put it equal y, then $y + \dfrac{1}{y} = 2\frac{1}{2}$, etc.

11. Use formula (4), p. 181, and factor by Ex. 7, p. 105 ;
$$x = -2 \pm \sqrt{3}.$$

12. $-\frac{1}{2} \pm \sqrt{\frac{69}{20}}.$ 13. $2 \pm \sqrt{2}$, and $x^2 - 6x + 6 = 0.$

14. $1 - p$, equation reduces to $(x + p - 1)\left(x^2 + x + \dfrac{1}{p - 1}\right) = 0.$

15. $7, 2$; if $x^2 - 9x + 18 = y$, equation is $y^2 + 2y = 24$, etc.

16. Divide by left member and clear. $\therefore 63(1 + x) = 62(1 - x)$, etc.

17. $a\left(1 \pm 2\sqrt{\dfrac{b}{ac}}\right)$, or 0 ; complete the division in left member, square, and 1 cancels.

18. $\frac{11}{12}$.   19. Sign before last fraction in left member should be $-$; equation reduces to $7x^2 - 34x + 9 = 0.$

20. Write $-$ for $+$ before 50 in denominator ; combine first pair fractions and second pair ; factor denominators and cancel ; then taking $x^2 - 5x = y$, $y^2 + 18y - 24 = 0$, etc.

21. $\frac{419}{110}$.   22. Combine first and last fractions, second and last but one, etc., and $2x + 7$ is found to be a factor ; then put $x^2 + 7x = y$, and resulting equation is $y^2 + 18y + 90 = 0$; $x = -3\frac{1}{2}$, etc.

23. Combine in pairs like numerators, then as in last example ; $2\frac{1}{2}$.
$$2\frac{1}{2} \pm \frac{1}{2}\sqrt{[5a + 13b + 17c \pm \sqrt{\frac{1}{4}(a - 3b - 2c)^2 + 12ab}}$$
$$\div (a + b + c)].$$

24. Combine first and second, etc. ; $-\frac{1}{2}(a + b), \infty$,
$$-\frac{1}{2}(a + b) \pm \frac{1}{2}\sqrt{\{\frac{1}{12}(a + b - 2c)^2 + \frac{1}{2}(a - b)^2\}}.$$

25. See Ex. 8 ; $n, \dfrac{1}{n}, -n \pm \sqrt{(n^2 - 1)}.$

EXERCISE LXXXI [a].   (Page 264.)

1. 11 or -24.   2. 26 and 19.   3. $\frac{4}{25}$ or $-6\frac{1}{4}$.

4. $\frac{1}{2}(\sqrt{5}-1)a$, $\frac{1}{2}(3-\sqrt{5})a$.   5. $16\frac{1}{2}$.

6. 50 coffee, 60 raisins.   7. \$240.

8. 100.   9. 12.   10. 11 vases.

11. A, 11 miles; B, 10.   12. 3, 4, and 5.

13. 25 cts.   14. 4d. a dozen.   15. 3 and 18.

16. 7.   17. A, 72 miles; B, 54 miles.

18. \$90 or \$10.   19. $\frac{2\frac{1}{2}}{5}$.   20. A, \$1800; B, \$1600.

21. \$3.   22. $\frac{1}{2}\{\sqrt{(2h^2-d^2)}+d\}$, $\frac{1}{2}\{\sqrt{(2h^2-d^2)}-d\}$.

23. 2.414 inches.

[b.]   (Page 266.)

1. 3 hours and 5 hours.   2. 36 and 30.   3. 63.

4. $-\frac{1}{4}(a+b)\pm\sqrt{\left(\dfrac{ab}{4m}+\dfrac{1}{16}(a+b)^2\right)}$.   5. 961.

6. 4200; read 780 in question.   7. 14 acres at \$75.

8. 10 seconds.   9. $5\frac{5}{8}$ miles.   10. 5 miles an hour.

11. 15 miles.

12. If x be cost and s selling price, then $x = s + \dfrac{x^2}{100}$; on solving
    it is seen that $4s$ cannot be greater than 100;
    see Art. 175 (i).

13. \$333$\frac{1}{3}$, \$666$\frac{2}{3}$.

14. 72, 12, 8; Let $x^2 =$ number remaining in smaller bag after
    handful is taken; then x^6 is left in *larger* bag, and $x^3 =$
    number in handful, and x^4 is number in larger after
    second lot is taken out; then $x^4 + x^2 = \frac{5}{3}(x^2 + x^3)$, and
    $x = 2$, etc.

15. If x represents per cent, then $620 = 82x + (3790 + 82x)\dfrac{x}{100}$.
    $x = 5$.

16. Let $2x =$ distance, then $\dfrac{x}{x-6} + \dfrac{x}{x-4} = \dfrac{6}{7} \cdot \dfrac{2x}{x-6}$; $x = 22$.

17. Let x = rate backwards, $4x$ = rate forwards, then

$$\frac{\frac{3}{4}}{4x} + \frac{\frac{1}{4}}{x}, \ i.\,e.\ \tfrac{7}{16}x = \frac{\frac{3}{4}}{4x+2} + \frac{\frac{1}{4}}{x-\frac{1}{5}}; \ \ 1 \text{ mile an hour.}$$

18. 90.　　**19.** 4900; x^2 being number of lines, equation is

$$\{\tfrac{49}{30}(x-10)\}^2 = 2\{x^2 - \tfrac{49}{30}(x-10)\}, \text{ or } 601x^2 - 20x \cdot 2254$$
$$= -100 \cdot 49 \cdot 43; \ \ x = 70.$$

20. A, half-past 4 o'clock; B, 5 o'clock.

EXERCISE LXXXII [a]. (PAGE 272.)

1. $x = \pm b$, $y = a \pm b$.　　　　2. $x = \pm b$, $y = a \mp b$.

3. $x = 3, -\tfrac{2}{5}, y = 1, -1\tfrac{4}{5}$.　　4. $x = 3, -2\tfrac{1}{2}, y = 3, -8$.

5. $x = 4, -10\tfrac{1}{4}, y = 5, -12\tfrac{13}{16}$.　6. $x = \pm 7, y = \pm 2$.

7. $x = \pm 20, y = \pm 16$.　　　8. $x = \pm 15, y = \pm 3$.

9. $x = 30, 10, y = 10, 30$.　　10. $x = 1, -5, y = -1, -7$.

11. $x = 3, -3\tfrac{1}{4}, y = 4, -3\tfrac{19}{28}$.　12. $x = 2, -\tfrac{1}{3}, y = 3, \tfrac{2}{5}$.

13. $x = 1, \tfrac{1}{7}, y = 2, 2\tfrac{4}{7}$.　　　14. $x = 10, 115, y = 6, -69$.

15. $x = 7, -4\tfrac{3}{4}, y = 3, -2\tfrac{7}{8}$.　16. $x = 1, -\tfrac{53}{81}, y = -4, 8\tfrac{11}{27}$.

17. $x = \pm 7, \pm 5, y = \pm 11, \pm 9$.　18. $x = 2, 3, y = 5, 4$.

19. $x = 2, 5, y = 6, 3$.　　　　20. $x = 5, \tfrac{3}{4}, y = 3, -1\tfrac{1}{4}$.

21. $x = 7, 1, y = 3, 9$.　　　　22. $x = \pm \sqrt{13}, y = \pm \sqrt{13}$.

23. $x = 3, y = 1$.　　　　　　24. $x = 2, \tfrac{5}{8}, y = -7, -\tfrac{1}{8}$.

.

[b.]

1. $x = \pm 5, y = \pm 1$.　　　　2. $x = \pm 11, y = \pm 2$.

3. $x = 0, \pm 2, y = \pm \sqrt{1\tfrac{1}{3}}, \pm 1$.　4. $x = 0, \pm 3, y = \pm 3, \pm 9$.

5. $x = \pm 3, \pm \tfrac{5}{3}, y = \pm 5, \pm 1\tfrac{3}{5}$.　6. $x = 0, \pm 1, y = \sqrt{\tfrac{7}{2}}, \pm 1$.

7. $x = \pm 2\tfrac{1}{8}, y = \pm \tfrac{1}{3}$.　　　8. $x = \pm 3\sqrt{\tfrac{3}{7}}, y = \pm \tfrac{1}{3}\sqrt{\tfrac{3}{7}}$.

9. $x = \pm 2\tfrac{1}{3}, \pm 1, y = \pm 1, \mp 3$.　10. $x = \pm 7, \pm 3, y = \pm 2, \pm 6$.

11. $x = \pm 1, \pm 1\tfrac{1}{2}, y = \pm 5, \mp 1\tfrac{2}{3}$.

12. $x = \pm a \div \sqrt{(a+b)}, y = \pm b \div \sqrt{(a+b)}$.

13. $x = \pm 6, y = \pm 2$.　　　　14. $x = \pm 3, \pm 8, y = \pm 5$.

15. $x = \pm 2, \pm \sqrt{\tfrac{2}{5}}, y = \pm \tfrac{1}{2}, \mp 2\sqrt{\tfrac{2}{5}}$.

16. $x = \pm 1, \pm 3\sqrt{3}, y = \pm 5, \pm \sqrt{3}$.

17. $x = 7, 4, y = 4, 7$.　　　　18. $x = 7, -5, y = 5, -7$.

19. $x = \pm 5, \pm 3, y = \pm 2, 7.$ 20. $x = \pm 5, \pm 4, y = \pm 3.$

21. $x = 4, y = 3.$ 22. $x = 14, 19, y = 19, 14.$

23. $x = 5, 4, y = 4, 5.$ 24. $x = 4, 2, y = 2, 4.$

25. $x = 4, y = 3.$ 26. $x = 3, 2, y = 2, 3.$

27. $x = 13, 9, y = 9, 13.$ 28. $x = 7, 4, y = 4, 7.$

29. $x = 3, 1, y = 1, 3.$ 30. $x = 3, 2, y = 2, 3.$

EXERCISE LXXXIII [a].   (PAGE 277.)

1. $x = 2, 1, y = -1, -2.$ 2. $x = 4, -3, y = -3, -10.$

3. $x = \pm \frac{2}{3}, \mp \frac{1}{3}, y = \frac{4}{3}, 0.$ 4. $x = \pm 1, y = \pm 7.$

5. $x = \pm 5, y = \pm 2.$ 6. $x = \pm 1, y = \pm \frac{1}{2}.$

7. $x = \frac{1}{2}(a \pm 2b), \; y = \frac{1}{2}(a \mp 2b).$

8. $x = \pm (a + b), \; y = \pm (a - b).$

9. $x = \pm \dfrac{a^2 + b^2}{a - b}, \; y = \pm \dfrac{2ab}{a - b}.$

10. $x = \frac{1}{2}, y = \frac{1}{3}.$ 11. $x = 6\frac{1}{2}, y = 1\frac{1}{2}.$

12. $x = 0, a, y = a, 0.$ 13. $x = \pm 9, \pm 5, y = \pm 5, \pm 9.$

14. $x = \pm 7, \pm 3, y = \pm 3, \pm 7.$

15. $x = \pm 4, \pm 3, y = \pm 3, \pm 4.$

16. $x = 2, -1, y = 1, -2.$ 17. $x = 11, y = 9.$

18. $x = -2, -3, 3 \pm \frac{1}{2}\sqrt{56}, \; y = -3, -2, 3 \mp \frac{1}{2}\sqrt{56}.$

19. $x = 5, -1, \frac{1}{2}(\pm \sqrt{41} + 5), \; y = 1, -5, \frac{1}{2}(\pm \sqrt{41} - 5).$

20. Treat $x + y$ as the unknown; $x = \frac{1}{2}\{a \pm \sqrt{(a^2 - 48)}\},$
    $y = \frac{1}{2}\{a \mp \sqrt{(a^2 - 48)}\},$ where $a = \frac{1}{2}(-3 \pm \sqrt{853}).$

21. $x = \frac{1}{2}(9 \pm \sqrt{-47}, \; y = \frac{1}{2}(9 \mp \sqrt{-47}).$

22. $x = 5, -2, -\frac{1}{2}(1 \pm \sqrt{41}), \; y = 2, -5.$

23. $x = 17, -6, \pm \sqrt{(118)} - 4, \; y = 3, -8\frac{1}{2}, 2 \pm \frac{1}{2}\sqrt{118}.$

24. $x = 5, 6, 7, 8, \; y = 8, 7, 6, 5.$

25. $x = \pm 13\sqrt{\tfrac{45}{218}}, y = \pm 7\sqrt{\tfrac{45}{218}}.$ 26. $x = 4, 2, y = 2, 4.$

27. $x = 1, 10, y = 10, 1.$ 28. $x = 3, 2, y = 2, 3.$

29. $x = 8, 4, y = 4, 8.$ 30. $x = 1, 1\frac{7}{17}, y = 2, -\frac{1}{17}.$

31. $(a'c - ac')^2 + (ab' - a'b)(b'c - bc') = 0.$

EXERCISE LXXXIV. (Page 281.)

1. $x = 8, 2, \quad y = 4, \quad z = 2, 8.$ 2. $x = \pm 4, \pm 9, \quad y = \pm 6$, etc.

3. $x = 7, \quad y = 6, \quad z = 5.$ 4. $x = 1, \quad y = 2, \quad z = 3.$

5. $x = \frac{4}{5}, \quad y = \frac{4}{3}, \quad z = 4.$

6. $x = \pm \frac{4}{3}\sqrt{3}, \quad y = \pm \sqrt{3}, \quad z = \pm 2\sqrt{3}.$

7. $x = 4, -7, \quad y = 3, -8, z = 6, 28, -2z$ in text.

8. $x = 1, 9, \quad y = \pm 4, \quad z = 2, -6.$

9. $x = 2, -14, \quad y = 3, -15, \quad z = 4, -16.$

10. $x = 1, \quad y = -2, \quad z = 4.$ 11. $x = 4, \quad y = -5, \quad z = 7.$

12. $x = 2, 7, \quad y = 3, \quad z = 7, 2.$ 13. $x = y = z = \pm 1 \div \sqrt{2}.$

14. $x = 2abc \div (ac + bc - ab), \quad y, z$ symmetrical.

15. $x = \pm a^2 \div \sqrt{(a^2 + b^2 + c^2)}, \quad y, z$ by symmetry.

16. $x = \pm \sqrt{\{(a + b - c)(a + c - b) \div 2(b + c - a)\}},$
    y, z by symmetry.

17. $x = abc \div (ab + ac - bc), \quad y, z$ by symmetry.

18. $x = \sqrt{\{(c + a - b)(a + b - c) \div (b + c - a)\}}.$

19. Add n^2 to each equation and factor :
    $x = -n \pm (a + n)(c + n) \div (b + n), \quad y, z$ by symmetry.

20. $x = \pm a \sqrt{\{(b + c - a) \div [(a + b - c)(a - b + c)]\}},$
    y, z by symmetry.

21. $x = (bc + ca + ab) \div a, 0 ; \quad y, z$ by symmetry.

22. $x = 1, 2, 4, \quad y = 2, 4, 1, \quad z = 4, 1, 2.$

23. $x = \pm (-m + n + p) \div \sqrt{2(m + n + p)} ; \quad y, z$ by symmetry.

24. $x = \pm (-bc + ca + ab) \div \sqrt{(2abc)} ; \quad y, z$ by symmetry.

25. $x = 0$, or $\pm 1 \div (c - a) ; \quad y, z$ by symmetry.

26. $x = \sqrt[3]{(b^2 c^2 \div a)} ; \quad y, z$ by symmetry.

EXERCISE LXXXV [a]. (Page 285.)

1. $\frac{1}{2}(1 + \sqrt{5}), \frac{1}{2}(3 + \sqrt{5}).$ 2. $\frac{1}{2} \pm \frac{1}{6}\sqrt{2193}, -\frac{1}{2} \pm \frac{1}{6}\sqrt{2193}.$

3. $36, 16,$ or $-36, -16.$ 4. $\pm \sqrt{(pq)}, \pm \sqrt{(p \div q)}.$

5. $20.$ 6. $\pm \frac{1}{2}(p + q)\sqrt{(a \div pq)}, \pm \frac{1}{2}(p - q)\sqrt{(a \div pq)}.$

7. $7, 21, 35.$ 8. $343, 64.$

9. $\frac{5}{7}, \dfrac{-1\frac{1}{2}}{\frac{1}{2}}.$ 10. $36.$

11. $34, 17, 51,$ or $-204, 612, -306.$ 12. $36.$

13. $\sqrt[3]{119} \pm \frac{1}{2}\sqrt{6} \div \sqrt{119}$.   Add and subtract the equations.

14. 8 ft., 10 ft.   15. 88 yds., 55 yds.

16. 63 ft., 45 ft.   17. 20 m., 30 m.

18. 6%, 7%.   19. 102 from length, 114 to width.

20. 18, 9·6.   21. 100 at $75 each.

22. 13, 10.   23. A 40 at $1.20, B 30 at $1.60.

24. 3, 5, 10.   25. 3, 4, 5.

[b.]

1. Edges $(x,\ y,\ z)$ are 1, 2, 4 :
    $x + y + z = 7,\ x^2 + y^2 + z^2 = 21,\ x^3 + y^3 + z^3 = 73$.
    Cube first equation by formula H (3), p. 85, and substitute
    from third, second, and square of first.

2. 864.   3. 2, 5, 8. See Ex. LXXXV, 16.   Add first two
    equations and subtract third, then symmetry.

4. 76 ; the *one digit* remainder is of course 9.

5. $2145, $2\frac{1}{2}$ years.   6. 4, 7, 10.

7. 290 yds.   8. 48, 10.

9. 8, 9, 10 ; see Ex. 4, p. 284.

10. 342 ; in last line of problem read 29.

11. 12, 4, 3.   12. 3, 6, 9. See LXXXIV, 19.

13. $\frac{1}{2}\{b \pm \sqrt{(c - a^2)}\} : \frac{1}{2}\{a \pm \sqrt{(c - b^2)}\} :: \frac{1}{2}\{a \mp \sqrt{(c - b^2)}\}$
    $: \frac{1}{2}\{b \mp \sqrt{(c - a^2)}\}$.

14. $x = \sqrt{\{(1 + a)(1 + b) \div (1 + c)\}}$, y and z by symmetry.
    See 4, p. 284.

15. 28 workmen, each 45 lbs., or 36 workmen and each 77 lbs. ;
    x = number of workmen, y lbs. carried each load, z number
    loads in one hour ; then $8xyz$ is whole weight moved, and
    $7(x + 8)(y - 5)z = 8xyz = 9(x - 8)(y + 11)z$.

16. $\dfrac{b - \sqrt{(b^2 - 4p)}}{a - \sqrt{(a^2 - 4p)}} = \dfrac{a + \sqrt{(a^2 - 4p)}}{b + \sqrt{(b^2 - 4p)}}$, where p = product of ex-
    tremes (or means), and $= (a^3 + b^3 - c) \div 3(a + b)$.

17. Of the first, 135, 62 ; of the second, 182, 57, (yds.).   18. 126.

EXERCISE LXXXVI [a]. (Page 296.)

1. $x^2 - 2x - 2 = 0$.   2. $2\frac{5}{4}$.

3. $32x^2 - 1412x - 23205 = 0$.   4. $x^2 - a^3b = 0$.

5. $p^2 - q$; $p(p^2 - 3q)$; $(p^2 - 2q) \div q^2$; see Art. 178.

6. $x^2 - (4a - 6b)x + 9a^2 - 10ab + 8b^2 = 0$.   7. $3 \cdot 14159$.

8. Positive for *all* values of x, expression $= (x - 2\frac{1}{2})^2$.   9. $7894\frac{9}{11}$.

10. $p - 2q + 3r$.   11. See Ex. 1, p. 294.

16. Assume $x^n = Ax + Bq$, then, since α, β are values of x,
     $\alpha^n = Ax + Bq$, and $\beta^n = A\beta + Bq$, whence A and B.

[b.]

2. $b^2 - ac = 0$.

4.   1. $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$;
     2. $q^2x^2 - (p^2 - 2q)x + 1 = 0$;
     3. $x^2 - (p^2 - q)x + q(p^2 - 2q) = 0$;
     4. $x^2 + \{p - \sqrt{(p^2 - 4q)}\}x - p\sqrt{(p^2 - 4q)} = 0$.

8. $cc' - aa' = 0$, $b'c + a'b = 0$.

[c.]

4. 579 and 135 are the roots of the first equation, 579 and -135 those of the second.

12. $4ab^2 + a'c - aa'c' = 0$; let roots of first equation be α, β, of second $\alpha + m$, $\beta + m$; form equations from relations of roots and coefficients and eliminate m.

13. (Right side of first equation should be 1.) Substitute for y in second equation, and apply condition of *equal roots* to resulting equation in x.

EXERCISE LXXXVII [a]. (Page 300.)

1. Min. 4.   2. Min. $-\frac{17}{4}$.   3. Max. $\frac{5}{4}$.

4. Min. $\frac{1}{18}$.   5. Min. $\frac{1}{50}$.   6. Min. $-\frac{1607}{2344}$.

7. Min. $\frac{7}{4}$.   8. Min. 2.   9. Min. $-\frac{1}{2}$, Max. $\frac{1}{2}$.

10. Max. 36 area, *i.e.* line is *bisected*.

[b.]

2. 81.   3. Min. $\frac{1}{2}a^2$, *i. e.* line is bisected.

4. $\frac{1}{2}a\sqrt{2}$, the sides are equal.   6. $(a+b)^2 \div 4ab$.

7. *All* numbers between $\frac{1}{3}$ and 3.   8. $\frac{1}{2}\sqrt{1}+\frac{1}{2}$.

11. $(b^2 - 4ac) \div a^2 = (n^2 - 4mr) \div m^2$.   12. $p = 6$, or $\frac{2}{3}$.

EXERCISE LXXXVIII [a]. (PAGE 305.)

1. **1.**   2. $24b^9$.   3. 5.   4. 1.

5. $\sqrt[3]{a^2}$; $2\sqrt[4]{a^3}$; $5a^3\sqrt{b}$; $7\sqrt{ab^3}$;

   $6\sqrt{a}\sqrt[4]{b}$; $\sqrt{a}\sqrt[3]{b}\sqrt[4]{c}$; $\sqrt[4]{a^3}\sqrt[7]{b^6}\sqrt[3]{c^{10}}$.

6. $a^{\frac{1}{2}}$; $a^{\frac{2}{3}}b^{\frac{4}{3}}$; $a^{\frac{1}{2}}b^{\frac{3}{4}}c^{\frac{1}{5}}$; $a^{\frac{2}{7}}b^{-\frac{3}{7}}c^{\frac{1}{7}}$; $a^{\frac{6}{5}}b^{\frac{2}{5}}c^{-\frac{1}{15}}d^{\frac{1}{20}}$.

7. *1.* a^2b^{-3}; $a^{-1}b^{-1}c$; $a^{\frac{1}{2}}b^{-2}$; $7a^2b^{\frac{3}{4}}$; $a^{-2}b^{-2}$; a^2b^2;

   $5a^5b^7c^{-5}$; $6a^{\frac{5}{4}}b^{\frac{1}{6}}c^{-\frac{7}{8}}$.

   2. $\dfrac{1}{a^{-2}b^3}$; $\dfrac{1}{abc^{-1}}$; $\dfrac{1}{a^{-\frac{1}{2}}b^2}$; $\dfrac{7}{a^{-\frac{5}{2}}b^{-\frac{3}{4}}}$; $\dfrac{1}{a^2b^2}$; $\dfrac{1}{a^{-2}b^{-2}}$;

   $\dfrac{5}{a^{-5}b^{-7}c^5}$; $\dfrac{6}{a^{-\frac{5}{4}}b^{-\frac{1}{6}}c^{\frac{7}{8}}}$.

[b.]

1. $(a^2 - b^2)^n$; $(x+y)^{p-q}$; $(x-y)^n$.   2. $\frac{3}{2}a^{\frac{45}{4}}$; 1.

3. $x^{\frac{4}{3}} - 4x^{\frac{1}{2}}$; $a^{\frac{2}{3}}b - a^{\frac{5}{4}}b^2 + 3a^2b^{\frac{4}{5}}$;

   $ab^{\frac{8}{3}} + a^{\frac{3}{2}} - a^2b^{\frac{1}{3}} - a^{\frac{6}{5}}b^{\frac{4}{5}}$.

4. $a^{\frac{1}{3}}b^{\frac{4}{3}} - 5ab^{\frac{6}{5}} + a^{\frac{4}{3}}b^{\frac{4}{3}} - a^{\frac{14}{5}}b^{\frac{16}{5}}$; $a^{-\frac{11}{5}}$; $3y^{-\frac{2}{3}}$; $x^{\frac{43}{20}}y^{\frac{5}{2}}$.

5. $\dfrac{c}{a^2b^3} - \dfrac{a}{b^3} + \dfrac{b}{a^4c^3} - \dfrac{1}{ab^2c^3}$; $\dfrac{c}{a^{\frac{2}{3}}b^{\frac{1}{4}}} + \dfrac{b^{\frac{2}{3}}c^{\frac{1}{3}}}{a^{\frac{2}{3}}} + \dfrac{1}{b^{\frac{3}{4}}}$.

6. $\dfrac{c^3}{a^2b^3} + 2abc - \dfrac{3a^3}{b^2c^3} + a^2b^2c^2$; $\dfrac{x^2c^4}{a^3y^2} - \dfrac{a^{a+b}}{y^{a+b}} + \dfrac{y^4}{5x}$.

7. $\sqrt[5]{x^3} + \sqrt[3]{y^2} - \sqrt[5]{z^3}$; $\sqrt[3]{x^{-2}}\cdot\sqrt[4]{y^3}$; $21\sqrt{a^{-6}}$;

   $\dfrac{3\sqrt[3]{b^2}}{4\sqrt[4]{a^5}}$.

8. $\dfrac{\sqrt[3]{c}}{\sqrt[3]{a^3b^2}} + \dfrac{\sqrt{a^3}}{\sqrt[3]{b^2}} - \dfrac{\sqrt[3]{a^2}}{\sqrt[4]{b^3}} + \dfrac{\sqrt{b}}{\sqrt{a}}$;

$\sqrt[2n]{x}\cdot\sqrt{a^3}$; $\sqrt[12]{a^x}$; $\sqrt[n]{a^{11}}$.

9. 8; $\frac{1}{32}$; $\frac{2}{25}$; $\frac{1}{16}$; $\frac{1}{11}$; $\frac{1}{9}$.   10. $1024a$; $a \div 32$.

11. $\dfrac{625a^5}{384b^2}$; $a^{6x-6y}\cdot b^{10x-12y}\cdot c^{14x-16y}$.   12. $(7x-6y)^{\frac{4}{3}}$; $(5a-7b)^x$.

13. 244140625; 2.   14. 0.

[c.] (PAGE 306.)

1. $\dfrac{b^{\frac{2}{9}}}{a^{\frac{8}{9}}}$; $a^{\frac{3}{4}}b^{\frac{4}{3}}$; $\dfrac{81x^8}{16a^8}$; $x^{\frac{5}{12}}z$.   2. x^{2n-1}; $a^{\frac{5}{4}}b^{\frac{3}{8}}$; $\dfrac{1}{(x-y)^2}$.

3. $a^{14}b^{-23}$; $a^{\frac{1}{9}}b^{-\frac{4}{9}}$; $\left(\dfrac{x}{y}\right)^7$. 4. $a^2 b^{\frac{m2+n2}{mn}}$; $b^{\frac{m2-n2}{mn}}$; a^{11mnp}; $b^{m2\cdot mn}$.

5. $a^{-\frac{1}{4}}$; $a^{12}b^6$; $x^{-4}y^4$; $a^{\frac{m2+n2}{mn}}\cdot b^{\frac{m+n}{n}}$.   6. $a^6b^4c^2$; a^2b^6c.

7. $\left(\dfrac{a^{\frac{5}{2}}b^2}{c^{\frac{3}{2}}}\right)$; $-\dfrac{x^6}{y^6}$; $a^{4\frac{9}{3}\frac{3}{6}}b^{3\frac{3}{6}\frac{7}{6}}$.

EXERCISE LXXXIX [a]. (PAGE 309.)

1. x^{a+b+c}; x^{a2+a+1}; x^{m2n2}; $a^{\frac{m3}{m+1}}$.

2. 2^{n2}; $\frac{1}{9}$; $(a^{\frac{1}{n}}\cdot b^{-\frac{1}{m}})^{pq}$.

3. $x^{\frac{5}{2}} + x^4 + x^{\frac{7}{2}} + 2x^3 + x^{\frac{5}{2}} + x^2 + x^{\frac{3}{2}} + x + 1$; $x + y$.

4. $x^{\frac{3}{2}} - y^{\frac{3}{2}}$; $x^2 + y^2$.

5. $x^4 + 2x^2y^2 + y^4 - xy$; $x^2 - 2xy + y^2$.   6. $4a^2 - b^2$.

7. $2x^{2a} - 42 - 9x^a + 6x^{-2a} + 11x^{-a}$; $4x^{\frac{2m}{n}} - 9y^{\frac{2p}{q}}$.

8. $a^3 + a^{-3} - 2 - a^{\frac{4}{3}} - a^{-\frac{2}{3}} + 2a^{\frac{1}{3}}$.

9. $x^3 + 4x^2y^{\frac{2}{3}} - 4x^2y - 16xy^{\frac{4}{3}} + 16xy^{\frac{5}{3}} - 64y^2$.

10. $3 + 2x^{-\frac{n}{2}} + 2x^{\frac{n}{2}} + x^{-n} + x^n$; $a^{\frac{4}{3}}x^{\frac{4}{3}} - b^4$.

11. $x^{\frac{1}{3}} - y^{\frac{1}{3}}$; $x + y + x^{\frac{1}{3}}y^{\frac{2}{3}} + x^{\frac{2}{3}}y^{\frac{1}{3}}$.

12. $x^{\frac{4}{5}} - x^{\frac{3}{5}}y^{\frac{1}{5}} + x^{\frac{2}{5}}y^{\frac{2}{5}} - x^{\frac{1}{5}}y^{\frac{3}{5}} + y^{\frac{4}{5}}$; $8a^{-2} + 7a^{-1} + 6$.

13. $5b^{\frac{1}{2}} + 4b^{\frac{1}{6}} + 3b^{-\frac{1}{6}} + 2b^{-\frac{1}{2}}$.

14. $x^{-\frac{5}{3}} + x^{-\frac{4}{3}}y^{-\frac{1}{3}} + x^{-1}y^{-\frac{2}{3}} + x^{-\frac{2}{3}}y^{-1} + x^{-\frac{1}{3}}y^{-\frac{4}{3}} + y^{\frac{5}{3}}$;

$\qquad a^{-\frac{5}{2}} - 2a^{-2}b^{\frac{1}{3}} + 4a^{-\frac{3}{2}}b^{\frac{2}{3}} - 8a^{-1}b + 16a^{-\frac{1}{2}}b^{\frac{4}{3}} - 32b^{\frac{5}{3}}$.

15. $x^{\frac{4}{3}} - 4x + 10x^{\frac{2}{3}} - 16x^{\frac{1}{3}} + 19 - 16x^{-\frac{1}{3}} + 10x^{-\frac{2}{3}} - 4x^{-1} + x^{-\frac{4}{3}}$.

16. $(a^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})(x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})$;

$\qquad (x^{\frac{2}{3}} - 2x^{\frac{1}{3}}y^{-\frac{1}{3}} + 2y^{-\frac{2}{3}})(x^{\frac{2}{3}} + 2x^{\frac{1}{3}}y^{-\frac{1}{3}} + 2y^{-\frac{2}{3}})$.

17. $(x^{\frac{1}{2}} - 8)(x^{\frac{1}{2}} + 7)$; $(3x^{\frac{3}{4}} - y^{\frac{1}{2}})(3x^{\frac{3}{4}} + 2y^{\frac{1}{2}})$.

18. $(x - 1)(x - x^{\frac{1}{2}} + 1)$; $(3x^{\frac{1}{2}} - 2y^{\frac{1}{2}})(2x^{\frac{1}{2}} - 3y^{\frac{1}{2}})$.

19. $-a^{-1}(1 + b^{-1})$, b^{-1} in denominator ; $\left(\dfrac{a}{b}\right)^{\frac{1}{2}(m + 1)}$;

$$\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}} + c^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} - b^{\frac{1}{3}}c^{\frac{1}{3}} - c^{\frac{1}{3}}a^{\frac{1}{3}}}{a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}}.$$

20. $ab^{-1} + 1 + a^{-1}b$. 21. $a^{\frac{2}{3}} + 2a^{\frac{1}{3}}y^{\frac{1}{5}} + 3y^{\frac{2}{5}}$.

[b.] (Page 310.)

1. 0 ; expression $= (ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1}) - ab(ab^{-1} + a^{-1}b)$
$\quad + a^{-1}b^{-1}(a^{-1}b + ab^{-1}) = (ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1}) - (ab^{-1} + a^{-1}b)$
$\quad \times (ab - a^{-1}b^{-1}) = (ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1} - ab^{-1} - a^{-1}b) = 0$.

2. See last question.

3. For last term read $a^{\frac{5}{6}}$. *Ans.* $a^{\frac{1}{3}} - 2a^{\frac{1}{2}} + a^{\frac{5}{6}}$.

4. $(x^{47}y^{46})^{\frac{1}{6}}$.   5. $2x(3 + x^2) \div (1 + x)^3$.

6. $a^{\frac{4}{3}} - 2ab^{\frac{1}{2}} + 3a^{\frac{2}{3}}b - 2a^{\frac{1}{3}}b^{\frac{3}{2}} + b^2$; in divisor read $2a^{\frac{1}{3}}b^{\frac{1}{2}}$.

7. $x^4 + 2x^3 - 8x^2 - 6x - 1$.

8. $ax^3 - a^{\frac{1}{3}}x = a^{\frac{1}{3}}x(a^{\frac{2}{3}}x^2 - 1)$, and second factor of this is contained in the product of the other two quantities,
$\qquad \therefore \text{ L. C. M.} = a^{\frac{1}{3}}x(a^2x^6 - 1)$.

9. $ab^{-1} + 1 - \frac{1}{2}a^{-1}b$; in second term in text read b^2 for b^{-2}.

10. $\frac{7}{12}$; in first term read $a^{\frac{3}{2}}$.

11. $\dfrac{(x - a^{\frac{1}{2}})(x + a)}{x - a}$; in second term of numerator read a.

12. $\sqrt[12]{(x^5y^6)} \div \sqrt[5]{(a^2b^2)}$.

13. Expression $= a^{\frac{2}{3}}(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{2}} + b^{\frac{2}{3}}(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{2}} =$ etc.

14. Numerator $= x^2\left(x^{\frac{m-n}{n}} - x^{\frac{n-m}{n}}\right)$,

Denominator $=$ same factor $\times \left(x^{\frac{m}{n}} + x^{\frac{n}{m}}\right)$.

15. $a^{6x} + a^{5x-y} + a^{4x-2y} + a^{3x-3y} + a^{2x-4y} + a^{x-5y} + a^{-6y}$.

16. $abcde$; divisor $= d^{19}$ (say) $= d^{17} \times d^2$, apply Law IV.

17. $x^{\frac{1}{2}} + (1 - m)\, a^{\frac{1}{2}}x^{\frac{1}{4}} + a$.

18. Expression $= x^2 - y^2 - (y^{-2} - x^{-2}) = xy\,(xy^{-1} -- x^{-1}y)$
$- x^{-1}y^{-1}\,(xy^{-1} - x^{-1}y)$, etc.

19. $a^{\frac{1}{2}}x + 2$; second remainder is $- 5\,(ax^2 + 7a^{\frac{1}{2}}x + 10)$
$= -5\,(a^{\frac{1}{2}}x + 2)(a^{\frac{1}{2}}x + 5)$, and H. C. F. is $a^{\frac{1}{2}}x + 2$.

20. $(ax^2 - 1)\,(ax^2 + 1)\,(a^3x^6 - 1) = a^5x^{10} - a^3x^6 - a^2x^4 + 1$.

21. $1 - \frac{1}{64}a^{-3}x^6$.

22. $a^{\frac{1}{3}}x\,(a^{\frac{1}{3}}x - 1) \div (2a^{\frac{1}{3}}x - 1$; read x^3 in first term of numerator;
denominator $= (3a^{\frac{1}{3}}x + 1)\,(2a^{\frac{1}{3}}x - 1)$;
$(1 - a^{\frac{3}{2}}) \div \{a\,(5a + a^{\frac{1}{2}})\}$;
denominator $= a^{\frac{3}{2}}\,(a + a^{\frac{1}{2}})\,(5a + a^{\frac{1}{2}})$.

23. $x - y + z = -\left\{\dfrac{1}{(b-c)\,(c-a)} + \dfrac{1}{(a-b)\,(b-c)} + \dfrac{1}{(a-b)\,(c-a)}\right\}$
$= 0$;
Expression $= \dfrac{-(a - b)\,(b - c)\,(c - a)}{(a - b)\,(b - c)\,(a - c)} = 1$.

24. $(a + b)\dfrac{m^2 + 3n^2 + m - n}{2mn}$.

EXERCISE XC. (PAGE 313.)

1. $\sqrt[3]{x^2}$; $7\sqrt{(xy^3)}$; $5\sqrt[4]{(x^3y^7)}$; $6\sqrt[6]{(x^9y^4)}$; $\sqrt{(a^3b^4)}$.

2. $27^{\frac{1}{4}}$; $512^{\frac{1}{5}}$; $9^{-\frac{1}{3}}$; $(\frac{3}{2})^{\frac{1}{4}}$; $8^{\frac{1}{5}}$.

3. *1.* $-\frac{2}{3}(b^6)^{\frac{1}{3}}$; $\frac{3}{4}(a^6b^9)^{\frac{1}{3}}$; $(4^{-9})^{\frac{1}{3}}$; $\{(\frac{3}{10})^3\}^{\frac{1}{3}}$; $(a^3b^{-3}c^{-9})^{\frac{1}{3}}$.

2. $-\frac{2}{3}(b^{-8})^{-\frac{1}{4}}$; $\frac{3}{4}(a^{-8}b^{-12})^{-\frac{1}{4}}$; $(4^{12})^{-\frac{1}{4}}$; $\{(\frac{10}{3})^4\}^{-\frac{1}{4}}$; $(a^{-4}b^4c^{12})^{-\frac{1}{4}}$.

4. $\sqrt{(36)}$; $\sqrt[3]{(250)}$; $\sqrt{\frac{1}{2}}$; $\sqrt{6}$; $\sqrt[3]{(90)}$; $\sqrt[3]{(16)}$; $\sqrt[4]{\frac{1}{3}}$;
$\sqrt[3]{a^4}$; $\sqrt[4]{(a^2b^5)}$.

5. $\sqrt{(ab)}$; $\sqrt{\dfrac{a}{b}}$; $\sqrt[3]{\dfrac{a^2}{b^2}}$; $\sqrt{(6a^2x)}$; $\sqrt[3]{\left(\dfrac{16a^2}{81b^2}\right)}$; $\sqrt[3]{\left(\dfrac{16a}{3}\right)}$; $\sqrt{(a^2 - b^2)}$.

6. $\sqrt[n]{(a^n b)}$; $\sqrt[n]{(a^{n+1})}$; $\sqrt[n]{\{(a^2 - x^2)^n (a + x)\}}$; $\sqrt{\left(\dfrac{a + b}{c}\right)^3}$; $\sqrt[3]{\left(\dfrac{x - 3}{x + 4}\right)^2}$.

7. $3\sqrt{10}$; $5\sqrt{5}$; $3\sqrt[3]{5}$; $9\sqrt{6}$; $18\sqrt[3]{2}$; $\frac{3}{2}\sqrt[3]{12}$; $7\frac{1}{2}$.

8. $8\sqrt[3]{2}$; $6\sqrt[5]{48}$; $2\sqrt[4]{5}$; $2\sqrt[4]{3}$; $10\sqrt{3}$; 2; $2\sqrt[3]{18}$; 12; $ab\sqrt[n]{b}$.

9. $\frac{2}{3}\sqrt{2}$; $\frac{2}{27}\sqrt{2}$; $\frac{2}{27}\sqrt[5]{16}$; $ay^{\frac{3}{4}}$; $a\sqrt[n]{a^m}$; $a^2 x\sqrt[3]{(ax^2 - 1)}$.

10. $\frac{3}{2}\sqrt[3]{150}$; $\sqrt[4]{375}$; $a^{\frac{1}{2}}(x + 5)$; $(x + y)\sqrt[3]{(x - y)}$.

11. $(x - a)\sqrt[3]{\{(x + a)(x^2 - a^2)\}}$; $x^{\frac{1}{2}}(x + y)$; $2(a - b)\sqrt[3]{(ab)}$.

12. $10\sqrt{3}$, $\frac{7}{2}\sqrt{3}$, $\frac{9}{15}\sqrt{3}$, $\frac{1}{2}\sqrt{3}$, $\frac{1}{6}\sqrt{3}$.

13. $4^{\frac{1}{2}}, 3^{\frac{1}{2}}$; $8^{\frac{1}{3}}, 6^{\frac{1}{3}}$; $10{,}000^{\frac{1}{4}}, 1000^{\frac{1}{4}}$; $33^{\frac{1}{5}}, 32^{\frac{1}{5}}$; $80^{\frac{1}{3}}, 50^{\frac{1}{3}}$; $a^{\frac{4}{3}}, a^{\frac{3}{2}}$; $a^{\frac{4}{24}}, a^{\frac{3}{24}}$; $a^{\frac{49}{35}}, a^{\frac{25}{35}}$.

14. $2\sqrt[3]{3} = 24^{\frac{2}{6}}$; $3\sqrt{2} = 18^{\frac{3}{6}}$; $24^{\frac{2}{6}} = 576^{\frac{1}{6}}$; $18^{\frac{3}{6}} = 5832^{\frac{1}{6}}$; and $\frac{5}{2}\sqrt[4]{1} = (244\frac{9}{64})^{\frac{1}{6}}$.

EXERCISE XCI. (Page 317.)

1. $2\sqrt{2}$; $8\sqrt[3]{5}$. 2. $-12\frac{1}{2}\sqrt{3}$; $11\frac{2}{3}\sqrt[3]{9}$.

3. $60\sqrt{3}$; $80\sqrt{3}$; 24. 4. $6 - 5\sqrt{6}$; $6\sqrt{3} + 3\sqrt{30}$.

5. -32.

6. $\frac{1}{3}\sqrt{2} + \frac{1}{3}\sqrt{3} + 2\sqrt{5}$; $\frac{1}{3}\sqrt{6} + \frac{1}{2}\sqrt[6]{32} + \frac{1}{6}\sqrt[4]{120}$.

7. $\frac{3}{4}(\sqrt{7} + \sqrt{3})$; $\frac{1}{2}(7 + 3\sqrt{5})$.

8. $\frac{1}{2}(17 - 3\sqrt{5})$; $\frac{1}{4}(16 - 13\sqrt{2})$; $\frac{1}{11}(7\sqrt{14} - 13)$; $2a^2 - 1 + 2a\sqrt{(a^2 - 1)}$.

9. $288\sqrt[12]{72}$; see 4, p. 316.

10. $x + y + z + 2\sqrt{(xy)} - 2\sqrt{(xz)} - 2\sqrt{(yz)}$; $13x^2 + 4 + 12x\sqrt{(x^2 + 1)}$.

11. $\sqrt{x} - \sqrt{a}$; $x^{\frac{2}{3}} - (xa)^{\frac{1}{3}} - a^{\frac{2}{3}}$; $a - \sqrt{(ab)} + b$; $\frac{1}{79}(25 - 6\sqrt{2})(3 - \sqrt[4]{2})$.

12. Square and transpose radicals, square again, then
$$(ax + by + cz)^2 = 4 (abxy + bcyz + acxz)$$
$$+ 8 \sqrt{(abcxyz)} \{\sqrt{(ax)} + \sqrt{(by)} + \sqrt{(bz)}\}, \text{ etc.}$$

13. Rationalize.   14. 3.1003.   15. 3.160.

16. $(\sqrt{5} + 1) \{4 - \sqrt{(10 + 2 \sqrt{5})}\} \div 4$: $\{a + \sqrt{(a^2 - x^2)}\} \div x.$

17. $4x \sqrt{(x^2 - 1)}$: $1 \div (1 - x^2).$ 18. $2x^2 \div a^2.$

19. a; rationalize and substitute.

20. $\sqrt{(a - x)} \div (\sqrt{a} + \sqrt{x})$;   factor out $\sqrt{(a + x)}$ in denominator of first fraction and rationalize, resulting numerator cancels denominator of last fraction, etc.   20. $10\frac{2}{3}.$

EXERCISE XCII [a]. (PAGE 320.)

1. $\sqrt{3} + \sqrt{2}.$ 2. $\sqrt{11} + \sqrt{2}.$ 3. $\sqrt{10} - \sqrt{6}.$

4. $2 + \sqrt{2}.$ 5. $\sqrt{11} + \sqrt{5}.$ 6. $\sqrt{5} + \sqrt{2}.$

7. $\sqrt{6} + 1.$ 8. $2 + \sqrt{5}.$ 9. $2\sqrt{3} + 3\sqrt{5}.$

10. $\dfrac{\sqrt[4]{7}}{\sqrt{2}} (\sqrt{7} - \sqrt{3}.$ 11. $\sqrt{7} - \sqrt{3}.$

12. $3 - \sqrt{3}$; change 13 to 11.

13. $2\sqrt{5} - 3.$ 14. $3\sqrt{11} - \sqrt{41}.$ 15. $\sqrt{7} - \sqrt{2}.$

[b.]

1. $\sqrt[4]{3} (1 + \sqrt{2}).$ 2. $\sqrt[4]{5} (1 + \sqrt{2}).$ 3. $\sqrt[4]{2} (\sqrt{3} - \sqrt{2}).$

4. $5 + \sqrt{6}.$ 5. $\sqrt{51} - 7.$ 6. $\sqrt{17} + \sqrt{19}.$

7. $\sqrt[4]{6} (1 + \sqrt{2}).$ 8. $\sqrt[4]{2} (\sqrt{3} - \sqrt{2}).$ 9. $3\sqrt{5} + 6\sqrt{2}.$

10. $\frac{1}{2} (\sqrt{3} - \sqrt{\frac{2}{3}}).$ 11. $\sqrt{30} - \sqrt{\frac{1}{2}}.$ 12. $\sqrt[4]{2} + \sqrt[4]{\frac{1}{2}}.$

13. $\sqrt[4]{3} (1 + \sqrt{2}).$ 14. $\sqrt{(ab - ab^2)} - \sqrt{(ab^2)}.$

15. $\dfrac{\sqrt{1 + x} + \sqrt{1 - x}}{2}.$ 16. $3\sqrt{m} - 5\sqrt{n}.$

17. $\sqrt{(m^2 - n^2)} + n.$ 18. $\sqrt{(x + y)} + \sqrt{(x - y)}.$

19. $\sqrt{x + y} + \sqrt{z}.$ 20. $1 - x + \sqrt{1 + 2x - x^2}.$

21. $1 + \sqrt{2}$.   22. $\sqrt[4]{2}\,(\sqrt{5} + \sqrt{3})$; in text 60 under root sign.

23. $\sqrt{5} - 1$.   24. $\sqrt{5} + \frac{1}{2}\sqrt{3}$; 48 for 49 in text.

25. $\sqrt{3} - \sqrt{2}$.   26. $\frac{1}{2}(\sqrt{10} + \sqrt{2})$.

EXERCISE XCIII [a]. (Page 323.)

1. $a^2 \div 2$.   2. 8.   3. $\sqrt{5}$.   4. 21.

5. 9.   6. 2.   7. a^m.   8. 16.

9. -1.   10. a.   11. 25, $\frac{1}{25}$, read $5\frac{1}{5}$ in text.

12. 3.   13. 9.   14. 4 or $-14\frac{2}{3}$.   15. $7\frac{1}{4}$ or 4.

16. 363.   17. 9.   18. 2.   19. 2.

20. 5.   21. 6.   22. $3\frac{1}{2}$ or 16.   23. -243 or 32.

24. $\dfrac{a + b}{a - b}$.   25. 9.   26. $-1\frac{5}{16}$.

[b.]

1. $\dfrac{1}{b - 2}$.   2. $\dfrac{(a - b)^2}{2a}$.   3. -1.   4. 16.

5. $-(a + b)$.   6. 46.   7. 6.   8. $\frac{1}{3}$.

9. $\frac{1}{4}$. In text remove parentheses from denominator.

10. 27.   11. $\dfrac{(a - b)^2}{2a - b}$.

12. $\frac{1}{51}$.   13. ± 3; use $\sqrt{(x^2 + 7)}$ as the unknown.

14. 1.   15. $-1, 2, 3, 6$.   16. $\sqrt[4]{\frac{1}{4}(4a^2b^2 - b^4)}$.

[c.] (Page 324.)

1. $-2, 4, 6, 12$.

2. $0, \pm\sqrt{3}$; simplifies to $x\{\sqrt{(2 + x)} + \sqrt{(2 - x)}\}$

    $= \sqrt{2}\,\{2 + \sqrt{(4 - x^2)}\} = \dfrac{\sqrt{2}}{2}\,\{4 + 2\sqrt{(4 - x^2)}\}$

    $= \dfrac{\sqrt{2}}{2}\,\{\sqrt{(2 + x)} + \sqrt{(2 - x)}\}^2$; $\therefore \sqrt{(2 + x)} + \sqrt{(2 - x)}$

    is a factor, etc.

3. $0, (b^2 - 4a^2) \div 4a$.

4. $a\,(b - c) \div 2\sqrt{(bc)}$; see Ex. 5, p. 322.

5. $\dfrac{1}{a}\left(b - \dfrac{nc}{n-1}\right)^2$.

6. $0, \pm\sqrt{3}$.

7. $\frac{8}{5}$.

8. $81 \div a$.

9. $2a^2 \div (1 + a^2)$.

10. $-(a^2 + b^2) \div (a + b)$.

11. $(1 + 4b - 10b^2 + 4b^3 + b^4) \div (1 + b)^4$.

12. $\dfrac{1}{a}\sqrt{\left(\dfrac{2a}{b} - 1\right)}$; equation is $\sqrt{\left(\dfrac{1 + bx}{1 - bx}\right)} = \dfrac{1 + ax}{1 - ax}$; square and use formula (6), p. 181.

13. 30.   14. $\left\{\dfrac{a^2(c-d)^2 - b^2(c+d)^2}{2(c^2 + d^2)}\right\}^{\frac{1}{2}}$.   15. $\frac{7}{9}$.

16. $\left\{\dfrac{(c^3 - 2a)^3 + 27a^3c^3}{27c^3}\right\}^{\frac{1}{2}}$; cube by formula G (2), p. 85.

17. $\pm\frac{1}{2}\sqrt{\left\{1 - \left(\dfrac{c-2}{3c^{\frac{1}{3}}}\right)^3\right\}^{\frac{1}{2}}}$.

18. $-\frac{781}{1267}, -\frac{1023}{1025}$; divide by right member, and $\left(\dfrac{1 + x}{1 - x}\right)^{\frac{1}{5}} + \dfrac{3}{16}\left(\dfrac{1 - x}{1 + x}\right)^{\frac{1}{5}} = 1$, or $y + \dfrac{3}{16}\cdot\dfrac{1}{y} = 1$, etc.

EXERCISE XCIV [a].   (Page 327.)

1. $4x^3 - 3x^2 + 2x - 1$.

2. $8x^3 - 12x^2 + 6x - 1$.

3. $8x^3 - 12x^2y + 6xy^2 - y^3$.

4. $x^3 - 2x^2y + 2xy^2 - y^3$.

5. $2 - 3x - x^2 + 2x^3$.

6. $x^4 - 2x^3y + 3xy^3 - y^4$.

7. $1 + \frac{1}{2}a - \frac{1}{8}a^2 + \frac{1}{16}a^3 - \frac{5}{128}a^4$.

[b.]   (Page 328.)

1. $x^2 - x + 1$.

2. $2 - 4x + x^2$.

3. $1 + 3x - x^2$.

4. $y^2 - y + 2$.

5. $x^3 - x^2y + xy^2 + y^3$.

6. $a^2 - \frac{1}{3}a^{-1} + \frac{1}{3}a^{-2}$.

7. $1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3$.

[c.]

1. $1\frac{3}{4}$.   2. $1\frac{1}{2}$.   3. 10.   4. $-\frac{3}{4}$.

5. $(a^4 - d) \div (c - 2a^3)$.

6. $b \div (a)$.

7. 8.   8. 6.   9. $-\frac{1}{2}$.

10. $\pm 3a$.   11. $(m-n)^2 + a^2 = 0$.

12. Condition for square is $q^2 = 4p^2 \cdot (qr + q^2)$, etc.

13. See Ex. 3, p. 327; remainder in this case is $12ab^2x^2 - 24b^3$, which must $= 0$, etc.

14. $(a^{\frac{1}{3}}x + \frac{1}{3}a^{-\frac{2}{3}}b)^3 =$ given expression; expand and equate coefficients, $\frac{1}{3}a^{-1}b^2 = c$, $\frac{1}{27}a^{-2}b^3 = d$.